Theoretical Physics (Prof. Ruiz/UNCA) **Exam 1** **February 22, 2021**

Open Book/Notes/Internet (90 minutes including scan and email return).
All work and related steps must be explicitly shown for full credit.

[10] 1. Groups. A group $G=\{a,b,c,d,e\}$ with identity e. For any $a\in G$ and $b\in G$, the following is true: $abab^{-1}a^{-1}=e$. Prove that the group is abelian.

[25] 2. Integral. Use a derivative trick to evaluate $\int_0^\infty xe^{-ax}\sin(kx)\,dx$, where $a>0$, starting from the integral result $\int_0^\infty e^{-ax}\cos(kx)\,dx=\dfrac{a}{a^2+k^2}$. For 20 points max, you may instead evaluate $\int_0^\infty xe^{-ax}\cos(kx)\,dx$. Commit to one for official credit.

[25] 3. Waves. Show that $\psi(x,t)=Ae^{-i(ax+bt)}$ satisfies the wave equation with an associated auxiliary equation that relates a, b, and the velocity v. What is this equation, in simplest form, that relates a, b, and v?

[10] 4. E&M. If the second Maxwell equation was modified to be $\nabla\cdot\vec{B}=f(x,y,z)\neq 0$, explain in a sentence or two the new physics of this novel situation.

[30] 5. Gas and Work. A gas expands from point a to point b (see figure), where the pressure varies as $P=\dfrac{k}{V^2}$ during the expansion (k is a constant). What is the work done by the gas if the gas expands from $V_1=V_0$ to $V_2=2V_0$ as shown in the figure, i.e., from point a to point b?

Give your answer in terms of P_0 and V_0, where k does not appear in your answer.