Theoretical Physics **Final Exam** **December 6, 2011**
Prof. Ruiz, UNC Asheville

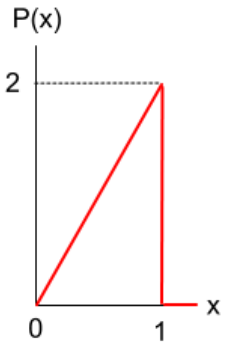
[5] 1. Groups and Matrices. Find the third matrix M_3 to go with the following to form a group where the binary operation is matrix multiplication.

$$M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad M_2 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

[10] 2. Relativity and Expanding. The relativistic velocity addition formula is $\beta = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2}$, where $\beta_1 = \frac{v_1}{c}$, $\beta_2 = \frac{v_2}{c}$, and $\beta = \frac{v}{c}$ (your relativistic sum of velocities 1 and 2). Give β to order $\frac{1}{c^3}$ when $\beta_1 = \frac{1}{10}$ and $\beta_2 = \frac{1}{10}$. What is the exact answer without doing the expansion to order $1/c^3$? Give your final answers as reduced fractions.

[5] 3. Statistics.

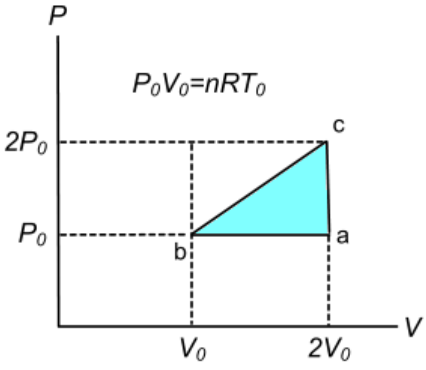
Find the average value of x for the probability distribution at the right. The function P(x) is zero when x > 1.



[15] 4. Engine. The engine illustrated has the cycle a-b-c. The gas is ideal: PV = nRT and U = 3nRT/2.

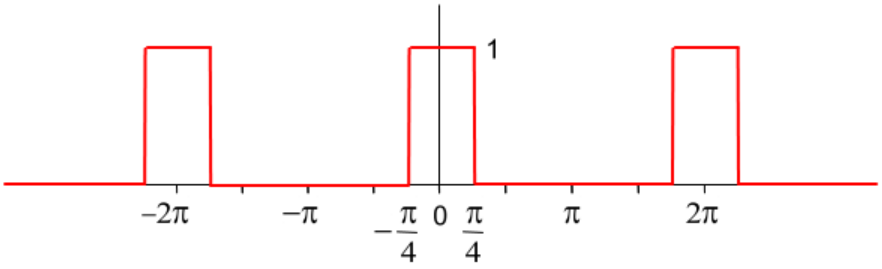
a. Let Q_{ab} represent the heat flow for the path a-b. A positive value means that heat flows into the gas; a negative indicates heat flows out. Simply state whether Q_{ab}, Q_{bc}, and Q_{ca} are negative, zero, or positive.

b. What is the efficiency of this engine if the heat that flows in during a cycle is $Q_{in} = 6nRT_0$?



[5] 5. States. There is a family of 7 people. How many ways can the following arrangement take place: 2 watching television, 3 having a snack in the kitchen, and 2 walking on a trail in the backyard.

[15] 6. Fourier Series. Find the Fourier Series for the periodic wave shown below. Your basic cycle for this repeating pattern is defined over our standard region $-\pi \le x \le \pi$, where the pulse is 1/4 the period (or wavelength) of the periodic wave.



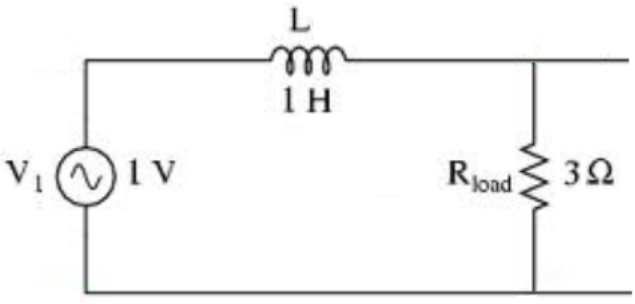
Pulse Train with a 25% Duty Cycle

Credit for this problem is heavily weighted on your explicitly writing out your answer by giving the first eight nonzero terms. You must start by writing "f(x) =" and then give the coefficients multiplied by the appropriate trig function for 8 nonzero terms, where each coefficient must be in simplest mathematical form with fractions and/or radicals.

[10] 7. Laplace Transforms. Use the real-imaginary trick to find the Laplace transforms of $\cos \omega t$ and $\sin \omega t$.

[10] 8. Convolution. Calculate $f * g$ where $f(t) = 1$ and $g(t) = t^2$.

[10]. 9. Transfer Function. A voltage $V_0 = \sin \omega t$ is applied to the LR circuit. The impedance of the inductor L is given by $Z_L = j\omega L$ and the impendence for the resistor is $Z_R = R$. Note $j = \sqrt{-1}$. Find the transfer function for this circuit. Then, find the magnitude of the output voltage when $\omega = 4$ with the values $L = 1$ and $R = 3$ given in the circuit. Is your filter low-pass or high-pass?



[15] 10. Complex Integration. Integrate $\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + 1}\, dx$ by first setting up an integral using the real-imaginary trick. Then, do the integral with complex integration techniques.