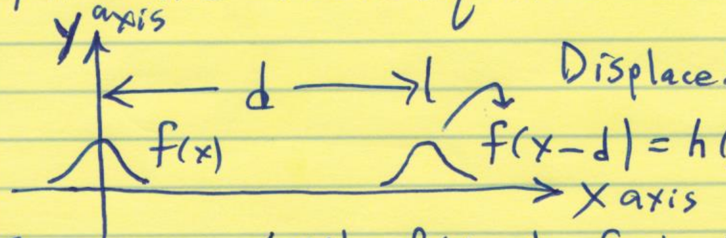
What Matters in College? See last page

January 30, 2020

F. Let There Be Light

F1. The Wave Equation



Displaced peak by distance d to right

Moving wave ↓ x-axis $d = vt$

$f(0) = \text{peak}$ $\quad h(d) = f(d-d) = f(0) = \text{Peak}$ $\quad \psi(x,t) = f(x-vt)$

Let $u = x - vt$ $\quad \dfrac{\partial u}{\partial x} = 1 \quad \dfrac{\partial u}{\partial t} = -v$

In search of a differential equation for $\psi(x,t)$

$$\frac{\partial \psi}{\partial x} = \frac{df}{du}\frac{\partial u}{\partial x} = \frac{df}{du}$$

$$\frac{\partial \psi}{\partial t} = \frac{df}{du}\frac{\partial u}{\partial t} = -v\frac{df}{du}$$

if $v \to -v$ you would need two different systems of eqs.

A second order Diff. Eq. has 2 solutions

We want one differential equation.

ψ_R traveling right solution

ψ_L " left "

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{d^2 f}{du^2} \qquad \frac{\partial^2 \psi}{\partial t^2} = \frac{d^2 f}{du^2}\underbrace{(-v)(-v)}_{v^2}$$

↳ Piece these together

Think units $x = vt$ for denominators

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

Wave equation in one dimension of space

Solution $\psi(x,t) = Af(x-vt) + Bg(x+vt)$

$f(x-vt)$

$g(x+vt)$ traveling down minus x-axis since $g(x+d)$ shifts things left

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

in 3D space

Recall $\nabla = \hat{i}\dfrac{\partial}{\partial x} + \hat{j}\dfrac{\partial}{\partial y} + \hat{k}\dfrac{\partial}{\partial z}$ $\quad \nabla^2 \equiv \nabla \cdot \nabla$ Laplacian

$$\boxed{\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}}$$

Wave Equation

F2. Let There Be Light

Maxwell Equations

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

For Free Space Equations ↔ No Sources

take $\rho = 0$

$\vec{J} = 0$

No charges

No Currents

Free Space Maxwell equations
$$\left\{\begin{array}{ll}\nabla\cdot\vec{E}=0 & \nabla\cdot\vec{B}=0\\ \nabla\times\vec{B}=\mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t} & \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}\end{array}\right.$$

Changing $\vec{B}$ field produces $\vec{E}$ field

Changing Electric field produces $\vec{B}$

The Secret

Play with the free space equations – explore

This exploration is Theoretical Physics

$$\nabla\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

Take time derivative since a second derivative in time is found in the wave equation.

Can move $\frac{\partial}{\partial t}$ through ∇

$$\nabla\times\frac{\partial\vec{B}}{\partial t}=\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$\hookrightarrow -\nabla\times\vec{E}$ Faraday's Law (see above)

$$\nabla\times(\nabla\times\vec{E})=-\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\nabla\times\vec{E}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ E_x & E_y & E_z\end{vmatrix}=\hat{i}\left(\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}\right)-\hat{j}\left(\frac{\partial E_z}{\partial x}-\frac{\partial E_x}{\partial z}\right)+\hat{k}\left(\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right)$$

$$\nabla\times(\nabla\times\vec{E})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z} & \frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x} & \frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\end{vmatrix}$$

$$[\nabla\times(\nabla\times\vec{E})]_x=\frac{\partial}{\partial y}\left(\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right)-\frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\right)$$

Can take derivatives in any order

$$=\frac{\partial^2 E_y}{\partial x\partial y}-\frac{\partial^2 E_x}{\partial y^2}-\frac{\partial^2 E_x}{\partial z^2}+\frac{\partial^2 E_z}{\partial x\partial z}$$

$$[\nabla\times(\nabla\times\vec{E})]_x=\frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}\right)-\frac{\partial^2 E_x}{\partial y^2}-\frac{\partial^2 E_x}{\partial z^2}$$

Add $0=\dfrac{\partial^2 E_x}{\partial x^2}-\dfrac{\partial^2 E_x}{\partial x^2}$ to right side

$$=\frac{\partial}{\partial x}\underbrace{\left(\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}\right)}_{\nabla\cdot\vec{E}}-\underbrace{\left(\frac{\partial^2 E_x}{\partial x^2}+\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}\right)}_{\nabla^2 E_x}$$

Add $\dfrac{\partial^2 E_x}{\partial x^2}=\dfrac{\partial}{\partial x}\dfrac{\partial E_x}{\partial x}$ Here

Subtracted Here

We discovered $[\nabla \times (\nabla \times \vec{E})]_x = \frac{\partial}{\partial x}(\nabla \cdot \vec{E}) - \nabla^2 E_x$

a Vector Identity $\rightarrow \nabla \times (\nabla \times \vec{E}) = \underset{\text{Gradient}}{\nabla}(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$ from before

But $\nabla \cdot \vec{E} = 0$ in free space

$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$

$$\boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$ Wave equation

Velocity c such that $\mu_0 \varepsilon_0 = \frac{1}{c^2}$

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

Light is Electromagnetic Wave

F3. Electromagnetic Waves

$$f, E \longleftrightarrow \lambda$$

γ X   UV $\overbrace{\text{V}}^{\text{VBGYOR}}$ IR   M   R

What Matters in College? by Astin 1997

Not the Budget
Not the Architect
Not the location
Not the library

1. Time on Task – Challenging Homework
   Golden Rule 3 hours outside class for every hour in class $\Rightarrow$ 9 hours/week
2. Access to Teacher
   RR0114 T, R 9:55–11:45
   W 9:30–3:15 ex lunch
   or RR0 124D
   Email: ruiz@unca.edu
3. Access to Peers – help each other { RR0119 (Hangout)

Note: Homework Grading (See Syllabus)
+4 for stating Problem/Steps/Comments
+4 for math/notation accuracy, +2 neatness

Complete the table below. The shorthand is powers-of-ten notation with the Metric System.

Number	**Decimal**	**Shorthand**	**Prefix**	**Abbr**
thousand	1000	10^3	kilo	k
million	1,000,000	10^6	Mega	M
billion	1,000,000,000	10^9	giga	G
trillion	1,000,000,000,000	10^{12}	tera	T
quadrillion	1,000,000,000,000,000	10^{15}	peta	P
hundredth	1/100	10^{-2}	centi	c
thousandth	1/1000	10^{-3}	milli	m
billionth	1/1,000,000,000	10^{-9}	nano	n

$|\leftarrow \lambda = 10\text{ m} \rightarrow|$

I-ARC. I=Inquiry: What is the wave speed v given λ and f?

A=Apply. Pick numbers to reason it out. Take the distance between peaks to be λ = 10 meters, and let five peaks go by each second, i.e., the frequency f = 5 Hz.

For your wave, what is the wave speed in meters per second? $10\text{ m} \cdot 5\,\frac{1}{s} = 50\,\frac{m}{s}$

R=Reflect. We can go deep now and write a general formula with symbols. $V = \lambda f$

The formula when applied to light is $c = \lambda f$. $c = 300{,}000\,\frac{km}{s} = 300{,}000{,}000\,\frac{m}{s}$

Also round off properly ← C=Communicate: Always include units with your numbers to be clear. How can the units for λ, f, and c lead you to the actual formula itself?

$$\left.\begin{array}{l}\lambda \Rightarrow \text{meters} \\ f \Rightarrow \dfrac{1}{\text{second}}\end{array}\right\} \quad \frac{\text{meters}}{\text{second}} \text{ is speed} \Rightarrow V = \lambda f$$

per second

Shake a charge and you get electromagnetic waves

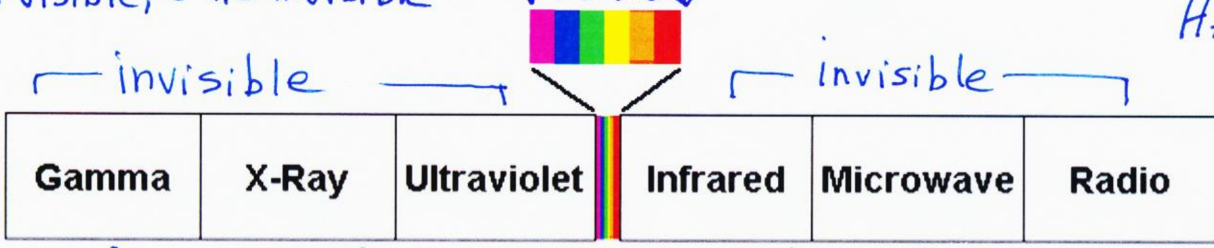
$\frac{1}{s}$ = per second

$\frac{1}{s}$ = hertz = Hz

7 regions of EM spectrum
1 is visible, 6 are invisible

400 nm ↓Visible↓ 700 nm

invisible — invisible



$f \rightarrow 10^{21}$ Zettahertz, 10^{18} exahertz, 10^{15} petahertz, 10^{12} terahertz, 10^{9} gigahertz, 10^{6} Hz megahertz

$\lambda \rightarrow 10^{-15}$ m, 10^{-10} m, 500 nm, 1 cm, 1 km

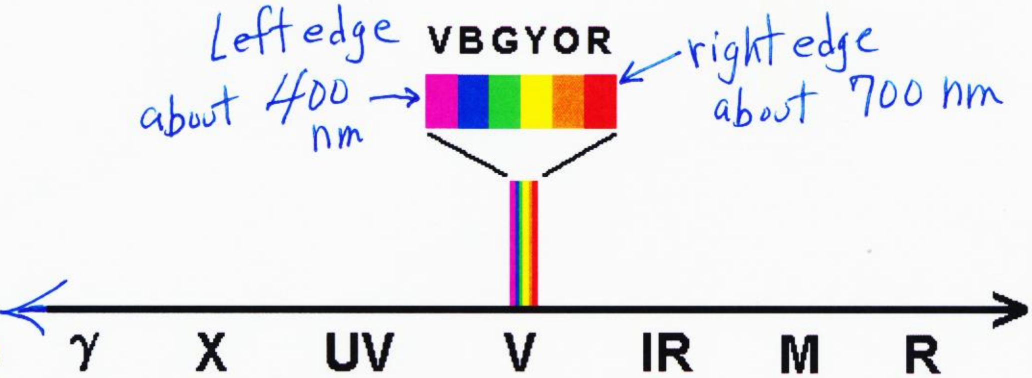
size → nucleus, atom, bacteria, marble, UNCA

The electromagnetic is given above in an idealized fashion. Add some frequencies, wavelengths, and sizes.

Below is a summary.

V = Violet, B = blue, G = green, Y = yellow, O = orange, R = red

Left edge about 400 nm → VBGYOR ← right edge about 700 nm



f increases to left

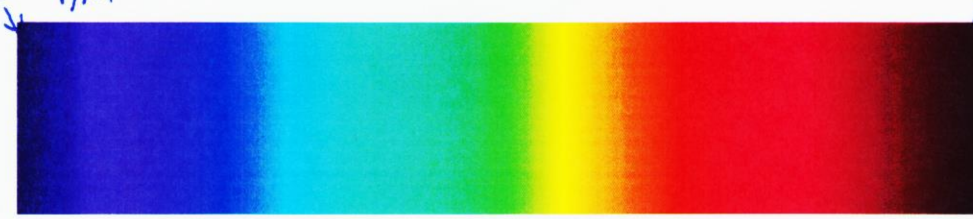
λ increases to right since $c = \lambda f$ (c fixed, a constant)

A more accurate visible part of the spectrum is given below; however, it is impossible to reproduce a precise spectrum using printer inks.

Visible Spectrum

400 nm

700 nm



Indicate the parts of the spectrum for the following.

1 EHz = 1 exahertz = 10^{18} Hz X-ray   1 ZHz = 1 zettahertz = 10^{21} Hz Gamma ray

1 μm = 1 micrometer IR   Hint: What is 1 μm in nanometers? 1000 nm

$1\,\mu m = 10^{-6}\,m = 1000 \cdot 10^{-9}\,m = 1000\,nm$ ⇒ beyond red in IR