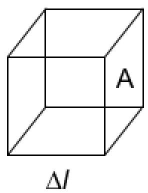
**Theoretical Physics**
**Prof. Ruiz, UNC Asheville**
**Chapter G Homework. Ideal Gas Law and Thermodynamics**

**HW-G1. Deriving PV = nRT for an Intro Course.** Consider a gas in a cube where each particle travels at the same speed v. Also, at any given time 1/6 of the particles are moving in the +x direction, 1/6 are moving in the –x direction, 1/6 are moving in the +y direction, 1/6 are moving in the – y direction, 1/6 are moving in the +z direction, and 1/6 are moving in the –z direction.




Let N represent the total number of particles in the box.

Analyze the change of momentum at the right wall and derive the ideal gas law by defining temperature in a similar way that we did in the chapter.

Now make your model more general by considering $N_1$ particles with velocity $v_1$, $N_2$ particles with velocity $v_2$, , etc. with $N = N_1 + N_2 + \ldots$ Keep the condition that 1/6 of the particles with velocity $v_i$ travels along one of the axis in a given direction. What result do you get now?

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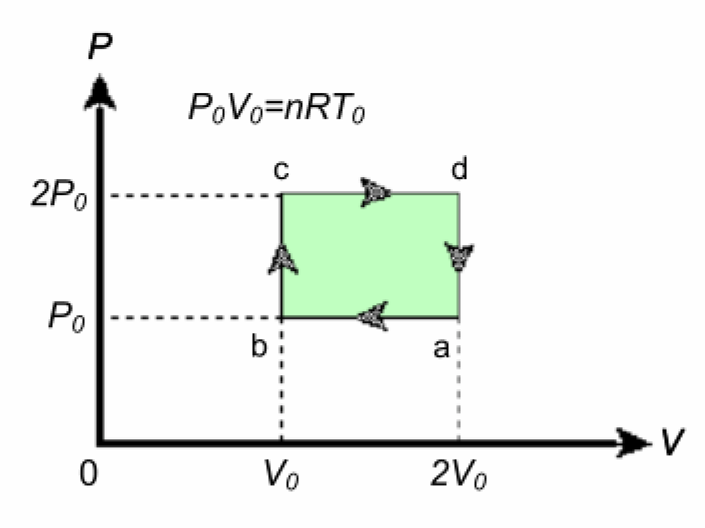
**HW-G2. A Simple Engine.** An ideal engine with an ideal gas, i.e., where $PV = nRT$, has the following cycle.




Compression Stroke: a to b
(compressing the gas fuel)

Ignition Phase: b to c
(with sudden pressure increase)

Expansion Phase: c to d
(engine does useful work)

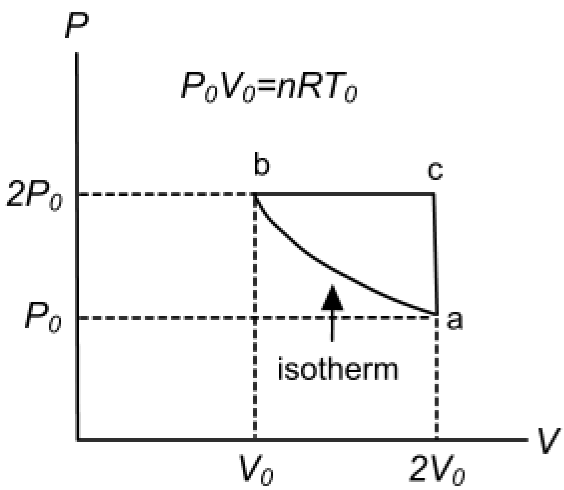
Pressure-Drop Phase: d to a
(returning to the initial PV point and ready to repeat the cycle)

Calculate the work for each of the four phases of the cycle: a-b (the isobaric compression), b-c (the isometric ignition), c-d (the isobaric expansion), and d-a (the isometric pressure drop). Then use the energy formula for an ideal gas $U = \frac{3}{2}nRT$ and the first law of thermodynamics $\Delta U = \Delta Q - P\Delta V$ to fill in the table below in terms of n, R, and $T_0$.

| | $\Delta U$ | $\Delta Q$ | $\Delta W$ |
|---|---|---|---|
| a to b | | | |
| b to c | | | |
| c to d | | | |
| d to a | | | |

Show that the efficiency: $\eta = \frac{W}{Q_{in}}$ is equal to $\eta = \frac{2}{13}$ for this system. Here, W stands for the net work performed and $Q_{in}$ is the input heat (heat that flowed into the system).

**HW-G3. Simple Engine With Isotherm.** An engine with an ideal gas follows the a-b-c cycle shown below.




First show that the pressure and volume at the endpoints "a" and "b" satisfy the equation that describes an isothermal process for an ideal gas.

Calculate the work for each of the three phases of the cycle:

a-b (the isothermal compression),
b-c (the isobaric expansion), and
c-a (the isometric pressure drop).

Fill in the requested quantities in a table like the one below, giving all entries in terms of n, R, and $T_0$.

Finally, calculate the efficiency of the engine to two significant figures.

| | $\Delta U$ | $\Delta Q$ | $\Delta W$ |
|---|---|---|---|
| a to b | | | |
| b to c | | | |
| c to a | | | |