Theoretical Physics
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Chapter F Homework. Differential Form for the Maxwell Equations

HW-F1. The Wave Equation. A short assignment illustrating the beauty and power of theoretical physics with short, elegant vector calculus. Use the differential form of the free-space Maxwell equations to derive the wave equation

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

for the electric field. But instead of doing it by tediously analyzing one component like we did in class, take the shortcut using the following vector identity:

$$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \ .$$

HINT: Start with the Maxwell equation $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ and apply the curl to both sides. You can move the curl inside the derivative with respect to time since space and time are independent.

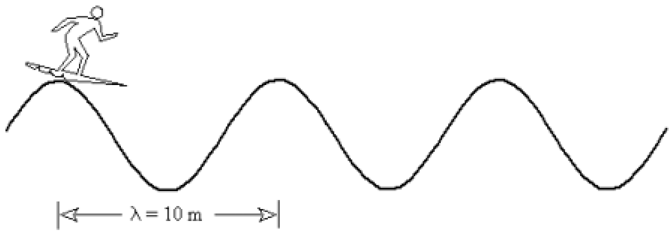
$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial (\nabla \times \vec{B})}{\partial t}$$

Simplify the left-hand side using the identity $\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$.

Note that a free-space Maxwell equation is $\nabla \cdot \vec{E} = 0$.

Then use the free-space Maxwell equation $\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ for the right side.

HW-F2. The Wave Relation. An ideal surfer rides on the crest of a wave.



a. If the waves are 10 meters apart and 5 crests go by second, what is the speed of the wave?

Let the distance from one crest to the next, your 10 m, be called the wavelength λ. Then let the number of crests per second be called the frequency f. Give the relation from your analysis that relates λ, f, and v (the speed of the wave). The common unit for frequency is hertz, where hertz = 1/s. i.e., per second. Explain why this must be the case for your units to come out correctly.

b. Now consider sound waves where v = 340 m/s at room temperature and pressure. What is the wavelength to the nearest centimeter (cm) for a 440-Hz sound wave, the note they use to tune an orchestra?

c. Now consider light waves where c = 300,000 km/s, where km is kilometer, i.e., 1000 meters. What is the wavelength to the nearest cm for a 30-GHz microwave? Note that the metric prefix G stands for Giga, which is equal to one billion.

d. What is the frequency in hertz to three significant figures for a 645-nm red diode laser? Note that the metric prefix n stands for nano, which is equal to one billionth. The problem requests you to give your answer to three significant figures. Remember, never give more significant figures than the most uncertain measurement. Trailing zeros typically do not count as significant figures, but the speed of light c = 300,000 km/s is actually accurate to four significant figures and good enough to use in this case. However, you are free to use overkill with c = 299,792 km/s. Overkill is good as long as you round off last to the proper number of significant figures.

IMPORTANT: Quick Review of Significant Figures and How to Handle Them

Suppose your answer from a calculation is 2.605948 after applying some formula to a hypothetical problem. This number has 7 significant digits as it stands. Then suppose when you started the problem, you used three input measured numbers for some formula in your calculation and these had significant figures 3, 4, and 6 respectively. Then, you should NOT report 2.605948 as your answer since you cannot be sure of that many digits. You should instead, round off your answer to three significant digits and report 2.61. If you include more digits in your answer because the calculator says so, then your answer is considered wrong as you cannot be sure of the accuracy of the digits to that extent. This is taught in intro chemistry courses and less so in intro physics courses. Since it is important to know, we take time here to explain it in case you did not take intro chemistry or physics.