Theoretical Physics
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Chapter C Homework. Relativity and Four Vectors

HW-C1. Time Dilation in GPS Satellites. Clocks on GPS satellites, when compared to stationary clocks on the Earth, run slower due to special relativity and faster due to general relativity. We will consider special relativity here. We can adapt our time dilation formula from class as

$$T_{Earth} = \frac{T_{Satellite}}{\sqrt{1-\frac{v^2}{c^2}}}\ .$$

The orbital period of a GPS satellite is one-half an Earth day with respect to the fixed stars, which is 11 hours and 58 minutes. But we will take our satellite to have a period of 12.00 hours. Note that when you specifically include zeros after the decimal point, we mean they are significant, i.e., not rounded off. Therefore, 12.00 has 4 significant figures, while 1200 would have 2. The latter has 2 since the actual value could be 1213 or 1226 since both round off to 1200. However, 1200. has 4 significant figures due to that decimal point. The actual value is now more restricted. It could be 1200.2 or 1199.6.

In Chapter 0 we derived Kepler's Third Law.

$$r^3 = \frac{GM}{4\pi^2}T^2$$

a) Use Kepler's Third Law to find in meters the distance r from the center of the Earth to a satellite that has a period T = 12.00 hours. Use values to at least 4 significant figures and report your answer to 4 significant figures. As an example, the mass of the Earth to 4 significant figures is M = 5.972 x 10^{24} kg. Box off your answer.

b) Take the orbit to be perfectly circular and find the satellite speed $v = \frac{2\pi r}{T}$ in meters per second to 4 significant figures. Box off your answer.

c) Now use the relativity time formula to calculate the time lost by the satellite clock in one day. Remember, the satellite clock runs slower (i.e., it loses time) compared to a stationary clock on Earth. Report your answer in microseconds to three significant figures. Box off your answer.

HW-C2. Perpendicular Relativistic Velocity Formula. Do Practice Problem PC3 in our text book (Perpendicular Relativistic Velocity Formula).