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Michael J. Ruiz and Lowell Hobbs



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Inharmonic Spectra with a Rock Guitar Effects Pedal

Michael J. Ruiz and Lowell Hobbs, University of North Carolina at Asheville, Asheville, NC

Students in introductory physics courses encounter harmonics when they study standing waves on strings and in pipes. The Fourier spectrum, which plots amplitude against harmonic number, can describe all periodic tones imaginable. However, most sounds are aperiodic and therefore have additional spectral components. A real-world application that includes frequencies outside the harmonic series can be found in guitar pedal effects used by rock musicians. A four-minute video¹ accompanying this paper includes the exotic sound of guitar playing when the guitar pedal adds inharmonic frequency components. The principle underlying the pedal effects discussed in this paper is balanced modulation. Some inharmonic spectra based on balanced modulation can easily be sketched after students have learned about timbre and Fourier spectra.

Background

This interdisciplinary paper gives the teacher an opportunity to connect frequency spectra to a real-world invention used in the world of rock music. Here the musician wants to extend the natural sound of the guitar by including strange effects to enhance the rock music experience. Students are sure to find this audio application of physics interesting.

In introductory physics, the frequency spectrum of a periodic tone, perceived as the timbre, is explained by Fourier's superposition of harmonics, where the n th harmonic is given by the familiar formula

$$f_n = nf_1. \quad (1)$$

The frequency f_1 is the fundamental and the other harmonics are the overtones. The timbre allows one to distinguish a violin from a flute when both instruments play the same pitch at the same loudness level.

Students typically first encounter Eq. (1) while studying the frequencies of standing waves on strings and open pipes. After standing waves, frequency spectra of periodic tones, often called Fourier spectra, are commonly discussed.²⁻⁶ Fourier plots of harmonic amplitudes for the frequency components of Eq. (1) against the harmonic number n are readily found in introductory texts for physical science majors,² life science majors,³⁻⁴ and non-science students.⁵⁻⁶

After students learn about harmonics and Fourier spectra, the teacher can ask students: What would a spectrum sound like with frequencies outside the set described by Eq. (1)? The answer is dissonant and unmelodious. Such a sound is often referred to as inharmonic since its frequency components are no longer in the harmonic series of Eq. (1).

Inharmonicity is present to a small degree in real strings. The ideal strings encountered in introductory physics are perfectly flexible with linear restoring forces. Their frequency

modes of vibration are given by Eq. (1). However, real strings have some stiffness, which introduces nonlinearity in the differential equation for the string. Fletcher and Rossing⁷ give the following more general formula in order to account for string stiffness:

$$f_n = nf_1^0 \sqrt{1 + Bn^2}. \quad (2)$$

The "frequency f_1^0 is the fundamental of the same string without stiffness"⁷ and B is a positive inharmonic coefficient fully discussed in Ref. 7. Equation (2) is especially relevant for the piano, where the strings are stiffer compared to other stringed instruments. Fletcher found⁸ for piano strings that B ranges on the order from 10^{-4} to 10^{-2} . The raised frequency of the second harmonic per Eq. (2) will beat with the fundamental of the note tuned an octave higher at exactly double the frequency. To work around the inharmonicity of piano strings, the piano tuner needs to stretch the tuning of octaves to avoid these beats. In contrast to the mild inharmonicity of stiff piano strings, guitar pedals can introduce wild inharmonics, as we shall see.

Guitar pedal effects

Since the golden age of rock in the 1960s, electric guitar musicians have used guitar pedals to add innovative audio effects to their performances.⁹ Pedals have been developed by engineers to add a variety of electronic effects such as echoes, modulations, and distortions. All of these devices introduce inharmonics. We will focus on the type that uses balanced modulation, appearing in the 1970s "from the world of analog synthesizer keyboards."⁹

An example of such a pedal device is the Moogerfooger MF-102, which originally appeared in the Moog synthesizer.¹⁰ This unit receives input from an electric guitar or other audio source and introduces a rich variety of inharmonic components. Sum and difference frequencies are produced from each sine harmonic (partial) of the input modulator wave (guitar) and an internally generated carrier sine wave in the pedal. Though there are many pedals on the market that accomplish this feat,¹¹ the Moogerfooger adds an additional low-frequency oscillator to vary the frequency of the carrier wave. The result is that the sum and difference frequencies can be periodically expanded and compressed. We discuss this feature in the section on the low-frequency oscillator and demonstrate it in our accompanying video.¹

Balanced modulation

In amplitude modulation, a signal (the modulator) modulates the amplitude of another signal (the carrier). The term "balanced" is included when the modulator and carrier waves share the same common zero-reference voltage, a convention

that dates back to the days of vacuum tubes.¹² One often refers to balanced modulation as ring modulation since the classic electronic circuit consists of four diodes arranged in a pattern resembling a ring.¹³

General introductory physics students can appreciate balanced modulation without delving into circuit analysis. The balanced modulator is a device that takes two input sine waves of frequencies f_1 and f_2 , and outputs sine waves at the sum and difference frequencies $f_1 + f_2$ and $|f_1 - f_2|$. The teacher who is interested in analyzing what's inside the balanced modulator can consult books on radio electronics.¹⁴⁻¹⁵ However, such analysis will be too advanced for introductory students unless they have sophisticated knowledge of circuits with diodes. Nevertheless, if the students know trigonometry, they can be shown that the balanced modulator multiplies input sine waves. The result, using cosine functions where $\omega = 2\pi f$, is

$$\sqrt{2} \cos(\omega_1 t) \sqrt{2} \cos(\omega_2 t) = \cos[(\omega_1 + \omega_2)t] + \cos[(\omega_1 - \omega_2)t]. \quad (3)$$

The factors $\sqrt{2}$ ensure that the output waves each have amplitude 1. The output sum and difference frequencies are readily apparent in Eq. (3). With sine functions the result is essentially the same but with phase shifts.

To appreciate the theory behind why sum and difference frequencies will sound very strange together, the teacher can remind students that the periodic waves of well-defined musical tones can be analyzed by harmonics, as discussed in introductory texts.²⁻⁶ Any such pitch at frequency f can be constructed with sine waves of frequencies $f, 2f, 3f, \dots$ with appropriate amplitudes and phases. These sine-wave components are the harmonics or partials, the first harmonic being the fundamental and the others the overtones. For this reason, such a periodic tone is said to be harmonic. On the other hand, the output of the balanced modulator sounds dissonant and strange since the resulting sum and difference frequencies in general do not have frequency ratios involving simple integers. Such inharmonic sounds often have a tone characteristic of a bell since “the dominant overtones of a bell are not harmonic in the way that stretched strings are.”¹⁶

The spectrum for balance modulating two sine waves

Consider a specific case where the input sine waves have frequencies $f_1 = 100$ Hz and $f_2 = 600$ Hz. The output frequencies are the sum and differences: $f_1 + f_2 = 700$ Hz and $|f_1 - f_2| = 500$ Hz. The spectrum is shown in Fig. 1, where relative amplitude is plotted against frequency. The input sine waves are represented by dashed vertical lines and the output sine waves are solid blue lines. The carrier is the sine wave generated internally in the guitar pedal and the modulator signal in this case is a sine wave fed into the guitar pedal from an audio oscillator. Note that both the carrier and modulator waves are suppressed. The output waves are the two sine waves labeled upper and lower sidebands. The sideband terminology comes from the field of radio communication where a signal in the audio range modulates a radio carrier wave and the modula-

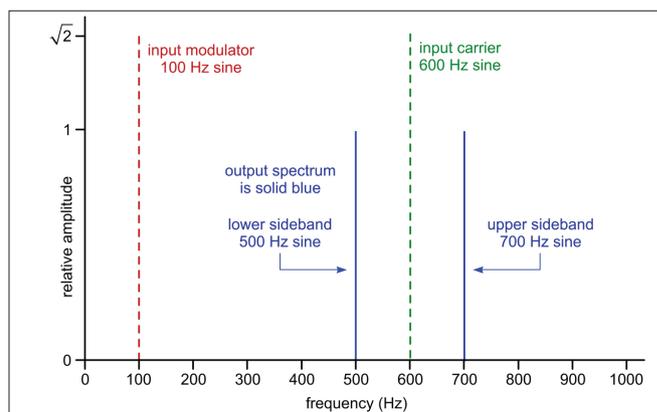


Fig. 1. A 100-Hz sine wave is balanced modulated with a 600-Hz sine wave, producing a 500-Hz output and a 700-Hz output, the difference and sum frequencies, respectively.

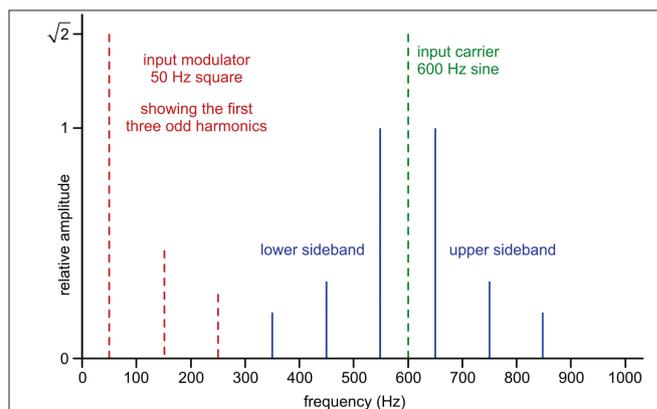


Fig. 2. A 50-Hz square wave (red) is balanced modulated with a 600-Hz (green) sine wave. The result due to the first few harmonics of the square wave are the very inharmonic spectral components in solid blue.

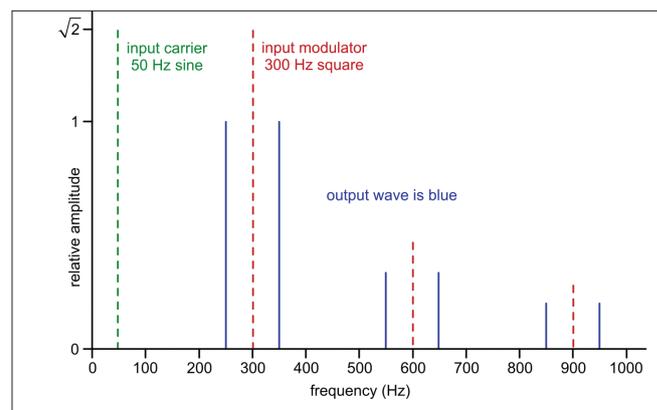


Fig. 3. An input 50-Hz sine wave (green) is balanced modulated with a 300-Hz square wave (red). The rich inharmonic output consists of split Fourier components of the original square wave.

tor information is mirrored in the two sidebands.¹⁷ We consider a square wave as the modulator in the next section.

Balance modulating a sine wave with a square wave

When one of the input waves is a square wave, the square wave needs to be decomposed into its sine wave components

since the balanced modulator rule must be applied to input sine waves. The Fourier spectrum for a 50-Hz square wave is shown in Fig. 2, where the first three odd harmonics are shown in red at frequencies 50 Hz, 150 Hz, and 250 Hz with relative amplitudes 1, 1/3, and 1/5, respectively. There are no even harmonics in the Fourier components of a square wave. Qualitative visual constructions of a square wave can be found in introductory texts^{2,6} and in a recently published Fourier HTML5 audio app with 16 harmonics.¹⁸

Adding and subtracting each input sine component with the 600-Hz carrier gives the very rich inharmonic output spectrum shown with blue solid lines. There is a mirror reflection symmetry about the carrier. Note how the spectral characteristics of the modulator are again present in these sidebands on either side of the carrier frequency. A figure similar to Fig. 2 was published by one of the authors (MJR) over three decades ago in a general paper on modular synthesizers¹⁹ in this journal.

When the input sine wave frequency is less than the input square wave frequency, the interesting output spectrum of Fig. 3 is found. In this case, a 50-Hz sine wave is balanced modulated with a 300-Hz square wave. The result is a very rich inharmonic output spectrum where each of the original Fourier components of the 300-Hz square wave is fractured into two. The spectrum of Fig. 3 is very inharmonic as the frequency ratios of the output spectral components now have more complicated relationships among themselves compared to those from the original square wave (red dashed vertical lines).

The mixer

In practice, an electronic input from any instrument, such as an electric guitar, modulates the carrier sine wave that is internally generated in the Moogerfooger. The mixer section of the pedal takes the output inharmonic sound and mixes back in the original guitar sound. This mixing is necessary in order not to totally lose the harmonicity of the guitar tones. Mixing in a little balanced modulation adds strange effects to the musical line. However, the performer can opt to output 100% modulation with no original guitar sound, making the musical line unrecognizable since then the harmonicity of the guitar is totally lost.¹

The low-frequency oscillator (LFO)

The Moogerfooger MF-102 includes a low frequency oscillator (LFO) that varies the carrier sine wave frequency during performance. Ask your students how this feature affects the spectral outputs of Figs. 2 and 3. In Fig. 2 the sidebands will roam. In Fig. 3 the sidebands will expand and contract, adding to the already strange quality of the output sound. Two types of carrier frequency modulation are provided: a sine wave and square wave. The sine wave produces a sinusoidal variation of the carrier while the square wave shifts the carrier abruptly between two frequencies. The user can set the frequency of the carrier modulation and the range of the sweep. The combinations are virtually limitless.

Our video

Students will become excited experiencing the Moogerfooger MF-102 in action. We include a video¹ where the wonderful features of this guitar pedal are demonstrated. We use three different input sources for three distinct demonstrations. First we input a 440-Hz sine wave into the Moogerfooger. This 440-Hz sine wave is then balanced modulated with the internally generated carrier sine wave of the pedal. We also demonstrate the LFO and its effects on the sound.

The second source is vocal, where we speak into a microphone connected to the guitar pedal. The result is a robotic voice. Finally, we use the guitar as the incoming audio signal. In practice, most performers place the Moogerfooger on the ground and toggle it on and off with a foot. We demonstrate the usual setting of the mixer where the original sound is gently modified by mixing in a little of the balanced modulation. At the end of our demonstration, we adjust the mixer so that the balanced modulation dominates. The original harmonicity is completely gone and the sound is exotically strange, full of inharmonicities. For students interested in learning more about the MF-102, the excellent manual¹⁰ is available online.

Conclusion

The theoretical analysis of balanced modulation is presented in a simplified manner. Spectra are given for three different input signals. These spectra can be introduced in general physics courses after the presentation of standing waves and Fourier's theorem. The material can also be used whenever nonlinear systems are discussed. In a power series, the first nonlinear term is the quadratic term. If one adds two sine waves and then squares the result, two of the resulting functions will have the sum and difference frequencies.

We use a guitar-effects pedal, the Moogerfooger MF-102, in a video¹ to illustrate the production of sum and difference frequencies. We perform demonstrations with three different modulators as input to the pedal device: a sine wave from an oscillator, speech from a microphone, and a signal from an electric guitar. Students will love the strange sounds from balanced modulation, a very cool interdisciplinary application of physics, where physics meets rock music.

Acknowledgment

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Michael J. Ruiz is professor of physics at the University of North Carolina at Asheville, USA. He received his PhD in theoretical physics from the University of Maryland, USA. His innovative courses with a strong online component aimed at general students have been featured on CNN. mjtruiz@gmail.com

Lowell Hobbs is a senior studying music technology at the University of North Carolina at Asheville, USA. Originally from the state of Maine (USA), he has lived in the Asheville area for the last nine years and enjoys the city, mountains, and playing guitar. Lowell is a guitarist and singer in his punk band, *Tongues of Fire*. lhobbs@unca.edu



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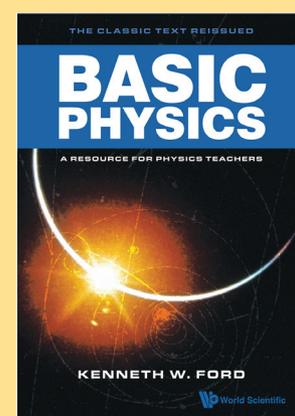
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