Clash of Harmonics in Stravinsky's *The Rite of Spring*

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This paper describes a fascinating connection between the physics of music and the famous chord in the dissonant rhythmic section of Stravinsky's *The Rite of Spring* (1913). The analysis of Stravinsky's chord will draw from the physics of harmonics, musical intervals, mathematics, and cognitive psychology. This highly interdisciplinary approach will especially appeal to students typically found in a physics course for non-majors. A video[1] accompanies this paper so that readers and students can hear the construction of Stravinsky's dissonant chord as well as an orchestra performing an excerpt of *The Rite of Spring*.

Bridging the gap between science and the humanities

Introductory physics students in general education science courses often have major interests outside of physics. Therefore, interdisciplinary ways to illustrate physics are very desirable in helping these students gain a deep appreciation of physics. *The Rite of Spring* (in French: *Le Sacre du printemps*) is also a ballet, with its premiere performance in 1913 choreographed by Vaslav Nijinksy. On the day of its 100th anniversary in 2013, an article in *BBC News*[2] stated, "Of all the scandals of the history of art, none is so scandalous as the one that took place on the evening of 29 May 1913 in Paris at the premiere of Stravinsky's ballet *The Rite of Spring*." Pulsating dissonance, striking departure from traditional musical ideas, and the theme of a pagan ritual all contributed to the negative reaction of many in the audience. But today, after decades of exciting movie music, your students will find the music cool.

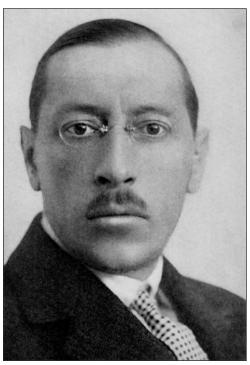

Fig. 1. Igor Stravinsky (1882-1971), a 1921 photo.[3]

See Fig. 1 for a photo[3] of Stravinsky taken eight years after *The Rite of Spring*. In his masterpiece, one dissonant chord has become the signature sound, accompanied by an unconventional rhythmic drive. This hypnotic chord is repeated 212 times in the section called the *Augurs of Spring*.[4] Reflecting on his work later, Stravinsky wrote,[5] "I was guided by no system whatever in *Le Sacre du printemps*… I had only my ear to help me. I heard and I wrote what I heard. I am the vessel through which *Le Sacre* passed." A musical creation emerging from artistic creativity without recourse to music theory presents an excellent opportunity to analyze the sound from the perspective of physics, the mathematics of musical intervals, and perceptual psychology. In this way, a gap between the humanities and science is bridged, as well as making a connection to social science.

The general idea behind the chord for *Le Sacre*

In the 1990s I acquired a deeper appreciation of the historical significance of *The Rite of Spring* through team-teaching in a core general education humanities course[6] at my institution. At the time, my son Evan was a student in the high school music division at one of our sister institutions, the University of North Carolina School of the Arts in Winston-Salem, studying orchestral composition. When I asked him about the Stravinsky work, he acknowledged its prominent place in a music history course he was taking. I told him that I wanted to incorporate this work in my course, "The Physics of Sound and Music," but I needed a physics angle.

So I taught him some basic open-pipe physics and he went off with the score to ponder the question. Shortly he ran back and exclaimed, "I got it!" He explained that the Stravinsky chord could be assembled by 1) using four harmonics from a pipe of a given length L, 2) taking four harmonics from a pipe with length $L + \Delta L$ such that the fundamental of the longer pipe is a semitone lower, and 3) separating each group of four harmonics by an octave. The details of these steps, including which harmonics are chosen and which are lowered, will be worked out after an important discussion on consonance and dissonance.

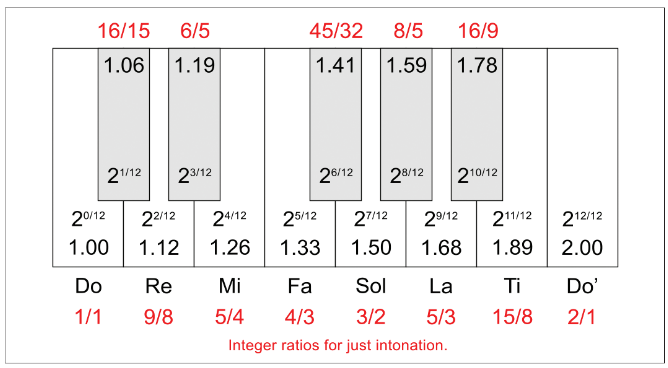

Fig. 2. The relative equal-tempered frequency ratios for the 12-tone chromatic scale with the integer ratios for just intonation taken from LoPresto.[7]

Consonance and dissonance

A knowledge of consonant and dissonant pairs of tones is necessary in order to analyze the discordant sound of the Stravinsky chord. Figure 2 presents the relative frequency ratios for equal temperament as well as a tuning based on integer ratios, where the integer ratios are taken from LoPresto.[7] Equal temperament fixes the interval from Do to Do′ (the octave) with a frequency ratio of 2:1. Since there are 12 semi-

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tone steps from Do to Do′, each adjacent note rising in pitch is found by multiplying the frequency of the lower pitch by the 12th root of 2, i.e., $\sqrt[12]{2} = 1.05946\ldots$. I use the analogy in class with investing money. What interest rate applied annually is needed for money to double in 12 years? The answer is 5.946 ...% = 6%, which corresponds to multiplying the money each year by the 12th root of two.

The motivation to use integer ratios in constructing scales dates back to "experiments attributed to Pythagoras," revealing "that consonant musical sounds relate to simple number ratios."[8] Standing waves on two strings produce a pleasing interval when the string lengths are in simple integer proportions. Scales constructed using relative frequencies with integer ratios are called "scales of just intonation" or simply "just scales." Centuries later, equal temperament was established because the integer ratios of a just scale are not preserved when the starting pitch (root) is moved to a different note, a process called "transposition" in music. However, equal temperament is a good compromise as can be seen by comparing the decimal values in Fig. 2 to their corresponding closest integer ratios. The merit of equal temperament is that all transposed scales maintain the same frequency ratios with respect to their starting notes. For excellent historical accounts of the physics of various scales from the Pythagorean scale to equal temperament, see Refs. 9 and 10.

Consonant musical intervals (pleasing-sounding pairs of two tones) have ratios of small integers such as 1:1 (sounding two Do tones, the unison), 2:1 (Do to Do′, the octave), 3:2 (Do to Sol, the fifth), and 4:3 (Do to Fa, the fourth). On the other hand, dissonant intervals have ratios with large integers such as 9:8 (Do to Re, the major second, M2), 15:8 (Do to Ti, the major seventh, M7), 16:15 (the semitone or minor second, m2), and 45:32 (known as the tritone). The abbreviations M2, M7, and m2 are included here because these interval designations will be used later.

Rasch and Plomp[11] distinguish between musical consonance and perceptual consonance. While musical consonance can be analyzed with frequency ratios[7-10] and depends on one's musical background, perceptual consonance is dependent on frequency separation.[11] Perceptual dissonance between two audible sine waves occurs "when the frequencies are within a *critical band* in which the ear has difficulty distinguishing between them."[7] Maximum dissonance is near the small frequency separation of the semitone. Since the semitone has an approximate 16:15 ratio, the ear must deal with a wave producing 16 cycles during the time frame when the other wave contributes 15 cycles. So musical dissonance has a basis in perceptual dissonance.[11] As we will see, Stravinsky's chord can be constructed from two harmonic series offset by a semitone; harmonic families afford much dissonance when both musical dissonance (large integer ratios) and perceptual dissonance (small frequency separation) are considered.

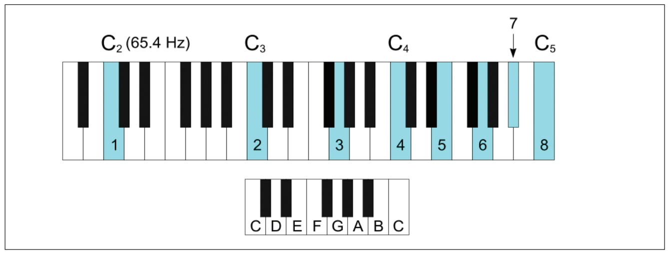

Fig. 3. The approximate positions of the first eight harmonics $f_n = nf_1$ for an open pipe (or string) with a fundamental f_1 = 65.4 Hz. Harmonics 1, 2, 4, and 8 fall precisely on their corresponding equal-tempered keys. The rest are approximate, with harmonic 7 being the worst.

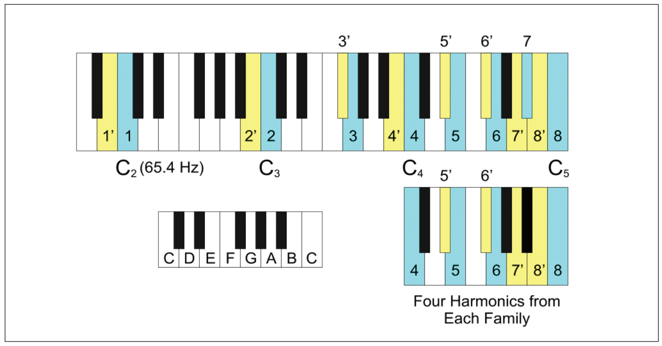

Fig. 4. Two families of harmonics (blue and yellow) separated by a semitone are shown in the upper part of the figure. In the lower right part of the figure, four harmonics are chosen from each family.

Constructing the Stravinsky chord

Harmonics emerge from the discrete patterns of standing waves that form on open pipes and strings. Physics teachers can first introduce students to several foundational class demonstrations[12] such as producing standing waves (harmonics) on a spring, twirling corrugated toy tubes to produce harmonics,[13] and dancing longitudinal waves.[14] Then, students will be prepared to apply harmonics in constructing the Stravinsky chord.

Figure 3 illustrates the approximate positions of the first eight harmonics $f_n = nf_1$ on an equal-tempered keyboard for an open pipe (or string) of the appropriate length L with a fundamental f_1 = 65.4 Hz, the second C on the piano. Each doubling of the fundamental (i.e., going up an octave) falls exactly on its corresponding equal-tempered key. Therefore, harmonics 1, 2, 4, and 8 are perfect matches. But be careful in class to say equal-tempered tuning rather than piano. The piano strings have a nonlinearity due to their stiffness and the second harmonic is slightly higher than double the frequency of the fundamental. Piano tuning is stretched a little so that the fundamental of the higher octave will not beat with the second harmonic of the lower octave.[15]

Figure 4 illustrates two families of harmonics, the blue family (same as Fig. 3) where $f_n = nf_1$, and the yellow family, one semitone lower, where $f'_n = nf'_1 = \frac{1}{2^{1/12}} nf_1$. The shift by one semitone (6% shift in frequency) makes the two families

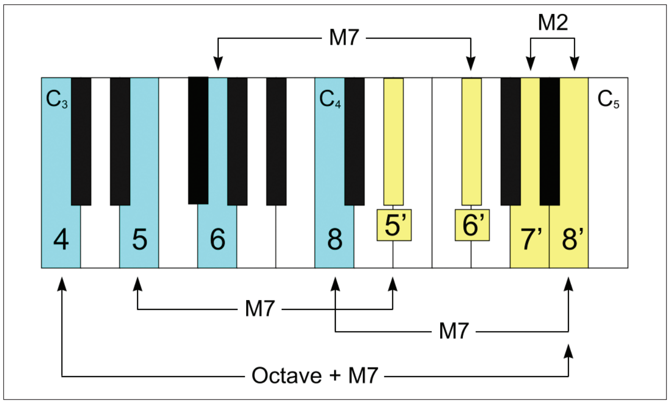

Fig. 5. The Stravinsky chord where the root is C. There are three major seventh intervals (M7), one major second (M2), and an interval consisting of an octave plus a major seventh.

extremely dissonant with respect to each other as discussed earlier. The yellow series comes from a pipe with length $L + \Delta L = 2^{1/12} L$ since the fundamental for the longer pipe is a semitone lower. The longer pipe is therefore 6% longer.

The Stravinsky chord consists of four tones from each family, where harmonics 4, 5, 6, and 8 are chosen from the "blue" family (unprimed family) and harmonics 5′, 6′, 7′, and 8′ are chosen from the "yellow" family (primed family). Neglecting the fact that when orchestrated, each of these pitches will be complex tones with their own series of harmonics, in the 8-note combination there are three semitone intervals (5′-5, 6′-6, and 8′-8), two major second intervals (6-7′ and 7′-8′), and one major seventh interval (4-8′). All six of these intervals are dissonant, from our section on consonance and dissonance.

For the orchestrated Stravinsky chord, the four harmonics from the "blue" family are lowered by one octave as illustrated in Fig. 5. The three semitone intervals of Fig. 4 now become major seventh intervals (M7). Considering the just interval ratios, recall that the very dissonant intervals include the semitone or minor second (m2, 16:15) as well as the major seventh (M7, 15:8) and major second (M2, 9:8). The dissonance of the major seventh (M7) can be thought of as arising since the major seventh just misses the pleasant octave (2:1) by being a semitone too short. The Stravinsky chord in Fig. 5 includes three major seventh intervals (M7) as well as a major second (M2) and an interval just falling short of the double octave (octave + M7).

The combination of these dissonant intervals makes the Stravinsky chord beautifully discordant. The actual chord used in *The Rite of Spring* starts on E_2 instead of C_3, but has the same relative frequencies as discussed above. A video[1] accompanies this paper so that readers can hear how the Stravinsky chord is put together and used in an excerpt from *The Rite of Spring*. The orchestral excerpt is performed by one of the orchestras at the Midwest Young Artists Conservatory in Illinois under the direction of its founder Maestro Dr. Allan Dennis.

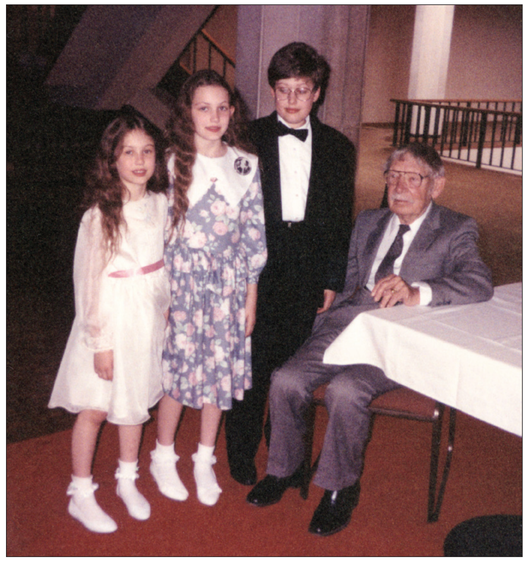

Fig. 6. The author's children and Igor Stravinsky's son Soulima (1993). Photo Credit: Candace Menedis.

Concluding remarks

The famous Stravinsky chord in *The Rite of Spring* has been discussed from a very interdisciplinary approach. The disciplines of music and history are at play as Stravinsky is an important modern composer within the historical context of music. Physics is used by starting with the harmonic series to understand how the Stravinsky chord can be constructed. Through psychoacoustics with its interdisciplinary connections to musical intervals, the biology of the ear, and cognitive neuroscience of perception psychology,[11] we are able to understand why the Stravinsky chord is dissonant. At each step of the way mathematics[10] plays an important role. Since we have worked within the framework of Western music theory and convention,[10] a sociological ingredient is present. People from other cultures with different musical notes and structures will not necessarily perceive consonances in the same way as Westerners.

Students also hear the Stravinsky excerpt, a masterful musical composition, adding a musical experience through the artistry of a performance. Teachers and students are encouraged to find YouTube performances with the ballet for which it was written, thus adding dance to the long list of interdisciplinary connections. Teachers can also incorporate a cinematic component by viewing the excerpt from the British TV movie "Riot at the Rite" (2005), where the "scandalous" 1913 premiere of *Le Sacre* (discussed in the opening section of this paper) is reenacted.

A final point is the author's personal connection to Igor Stravinsky (1882-1971). My children and Igor's son Soulima (1910-1994) hung out together (see Fig. 6 for a photo). My students pause to think twice here since Igor's son was born before my father was born. But waiting eight decades makes it all possible. My three children appeared with Soulima at the

1993 Stravinsky Awards International Piano Competition for young artists (ages 6 to 18) held at the University of Illinois at Urbana-Champaign, where Soulima was a piano faculty member for many years. In the photo from left to right are 1993 semifinalists Christa and Frances followed by finalist Evan, with Soulima Stravinsky at the table. Note that Evan is responsible for the initial physics analysis of this paper based on harmonics, which he figured out just a few years later as a teenager.

Acknowledgments

I would like to thank my son Evan for the analysis of the Stravinsky chord in terms of harmonics and family friend Candace Menedis for taking the photo of my children with Igor Stravinsky's son Soulima in 1993. Finally, I would like to thank Maestro Dr. Allan Dennis for permission to use an excerpt of his performance[1] of *The Rite of Spring* by one of his youth orchestras at his Midwest Young Artists Conservatory in Highwood, IL (mya.org).

References

1. M. J. Ruiz, "Video: The Stravinsky chord," (2017), http://mjtruiz.com/TPT/stravinsky/.
2. Ivan Hewett, "Did The Rite of Spring really spark a riot?" *BBC News Mag.* (May 29, 2013), http://www.bbc.com/news/magazine-22691267 , accessed May 31, 2017.
3. Photo by photographer Robert Regassi, frontispiece in *Miniature Essays: Igor Stravinsky* (J. & W. Chester, London, 1921). No author listed. Image from Wikimedia (Ras67, Bness2), digitally cleaned by the author of this paper.
4. Daniel K. L. Chua, "Rioting with Stravinsky: A particular analysis of The Rite of Spring," *Music Anal.* **26**, 59–109 (2007).
5. Igor Stravinsky and Robert Craft, *Expositions and Developments* (University of California Press, Berkeley and Los Angeles, 1981), pp. 147–148. First published in 1959.
6. Humanities 324 The Modern World, a team-taught core general education requirement, UNC Asheville, Asheville, NC, ever since the school became a four-year college in 1963.
7. Michael C. LoPresto, "Measuring musical consonance and dissonance," *Phys. Teach.* **53**, 225–229 (April 2015).
8. Neil Bibby, "Tuning and Temperament: Closing the Spiral," in *Music and Mathematics: From Pythagoras to Fractals*, edited by John Fauvel, Raymond Flood, and Robin Wilson (Oxford University Press, Oxford, 2003).
9. Dallin S. Durfee and John S. Colton, "The physics of musical scales: Theory and experiment," *Am. J. Phys.* **83**, 835–842 (Oct. 2015).
10. Gareth Loy, "Musical Scales, Tuning, and Intonation," in *Musimathics: The Mathematical Foundations of Music,* Vol. 1 (The MIT Press, Cambridge, MA, 2006), Chap. 3, pp. 39-96.
11. R. A. Rasch and R. Plomp, "The Perception of Musical Tones," in *The Psychology of Music*, edited by Diana Deutsch (Academic Press, New York, 1982.)
12. Gordon P. Ramsey, "Teaching physics with music," *Phys. Teach.* **53**, 415–418 (Oct. 2015).
13. Michael J. Ruiz, "The '2001' theme on the whistling tube," *Phys. Teach*. **54**, 310–311 (May 2016).
14. M. J. Ruiz, "Students dance longitudinal standing waves," *Phys. Educ.* **52**, 033006 (2017).
15. Richard E. Berg and David G. Stork, *The Physics of Sound,* 3rd ed. (Pearson Prentice Hall, Upper Saddle River, NJ, 2005).

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