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The monster sound pipe

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Abstract

Producing a deep bass tone by striking a large 3 m (10 ft) flexible corrugated drainage pipe immediately grabs student attention. The fundamental pitch of the corrugated tube is found to be a semitone lower than a non-corrugated smooth pipe of the same length. A video (https://youtu.be/FU7a9d7N60Y) of the demonstration is included, which illustrates how an Internet keyboard can be used to estimate the fundamental pitches of each pipe. Since both pipes have similar end corrections, the pitch discrepancy between the smooth pipe and drainage tube is due to the corrugations, which lower the speed of sound inside the flexible tube, dropping its pitch a semitone.

Background

Popular corrugated toy tubes that are twirled to obtain sounds have been around for decades [2–5]. The corrugated drainage pipe is a ‘monster’ version of the toy tube. One can spin the drainage tube outdoors and obtain a few overtones,[1] however the fundamental can be produced by tapping the tube [2]. The amusing outdoor demonstration to play overtones by swinging the tube around one’s head is included in the accompanying video [1]. However, the main focus here is on the in-class demonstration of tapping corrugated and smooth pipes in order to compare their fundamentals.

The Demonstration

One day at home, one of the authors (MJR) observed a landscape expert (David J Link) carrying a 10 ft (3 m) drainage pipe for use in directing water away from a rainspout. A nice deep bass tone was heard when David threw the drainage pipe to the ground. Slapping the drain pipe readily produced the same tone. It wasn't long before the demonstration described in this paper became a favorite at our school.

See figure 1 for a photo of the drainage tube in class. Shane and the instructor (MJR) hold each end as Valerie smacks the tube to obtain the fundamental. The note on the keyboard is located that best matches the tone. Pulling up an Internet keyboard is actually better since the students can then see all the keys clearly. Also, traditional keyboards are usually not available in physics classes. Other measurement methods can be explored such as

[1] Harmonics above the fundamental.

using an oscilloscope, bass guitar tuner, or a pitch-detection app on a smartphone. Before discussing the results of the demonstration, the basic theory of open-pipe tone generation is presented.

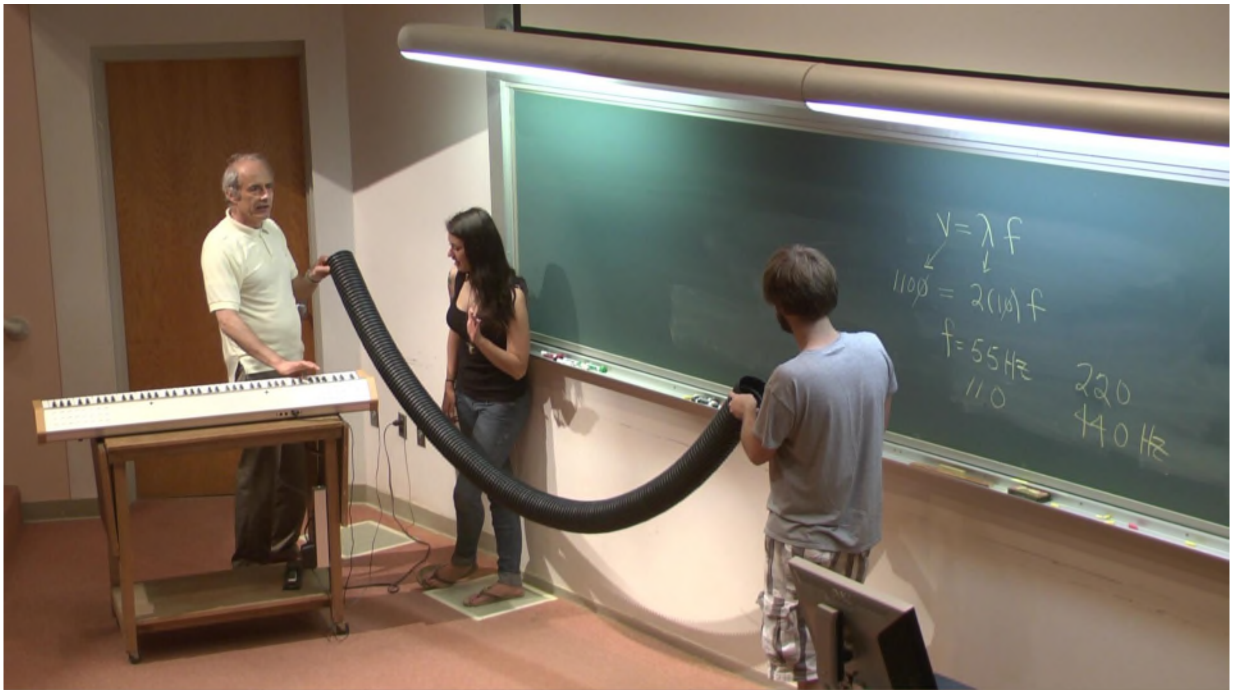

Figure 1. The 10 ft (3 m) drainage tube in class. The keyboard is used to match the frequency when the tube is smacked. The matched tone is found to be a semitone lower than the theoretical predicted value of A_1 = 55 Hz.

Basic open-pipe theory

A smooth open pipe of length L has a fundamental wavelength $\lambda = 2L$. Refer to figure 2 for an illustration of the fundamental standing wave in an open pipe where the sine shapes indicate the changes in pressure of the longitudinal wave along the pipe. Because the ends connect to the atmosphere where the pressure is constant (equal to atmospheric pressure), the end locations are pressure nodes. On the other hand, the pressure at the center of the pipe fluctuates in a sinusoidal fashion from a minimum (rarefaction) to a maximum (compression). This location where maximum changes in pressure occur is a pressure antinode.

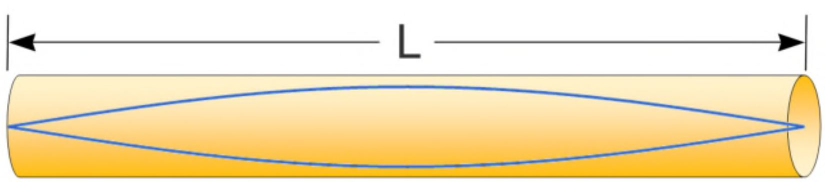

Figure 2. Fundamental standing wave in an open pipe. One half-wave, $\frac{\lambda}{2}$, fits in the pipe of length L so that the wavelength $\lambda = 2L$, i.e. twice the length of the pipe.

From the wave relation $v = \lambda f$, the frequency for the fundamental, i.e. the first harmonic, is

$$f = \frac{v}{\lambda} = \frac{v}{2L} \,. \quad (1)$$

In the units of the manufacturer, the pipe length is 10.0 ft and the speed of sound (to two significant figures) is 1100 ft s^{-1}, giving an estimated frequency

$$f = \frac{v}{2L} = \frac{1100 \text{ ft} \cdot \text{s}^{-1}}{2(10 \text{ ft})} = \frac{110}{2} = 55 \text{ Hz} \,. \quad (2)$$

Using British Imperial units, this calculation is mathematically easy enough to do without a calculator, allowing students to focus more intently on the physics. The simple math is an ideal feature when doing this demonstration for general students. The International System of Units (SI) can also be used with the approximate value 330 m s^{-1} for the speed of sound and 3 meters for the 10 ft pipe length in order to obtain a similar mathematical simplification,

$$f = \frac{v}{2L} = \frac{330 \text{ m} \cdot \text{s}^{-1}}{2(3 \text{ m})} = \frac{110}{2} = 55 \text{ Hz} \,. \quad (3)$$

This value brings us to our second fortuitous simplification – the piano key for this pitch can be precisely located on the keyboard.

Refer to figure 3 to see that middle A, the famous reference tone to which orchestras generally tune, is in the fourth octave on the piano. Middle A has a frequency $A_4 = 440 \text{ Hz}$. After dropping down an octave to $A_3 = 220 \text{ Hz}$, proceeding to drop by an octave two more times leads to $A_2 = 110 \text{ Hz}$ and $A_1 = 55 \text{ Hz}$ respectively. With a full-sized Internet keyboard [6] the pipe frequency can be determined experimentally by matching the tone produced by the pipe with the proper key struck on the keyboard.

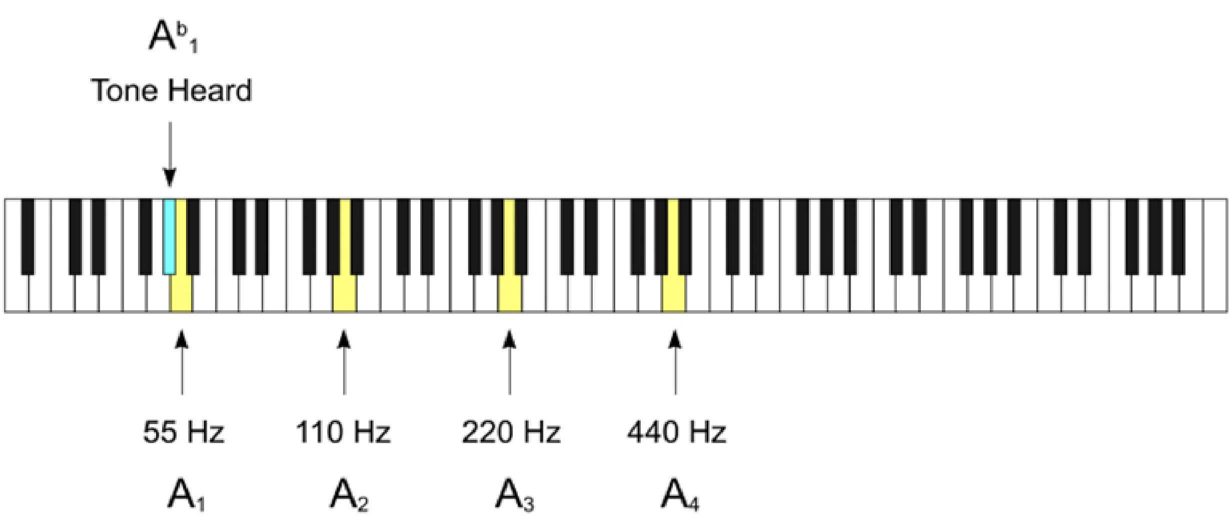

Figure 3. Piano keyboard showing concert A_4, three octaves below 440 Hz, and the note (A_1^b) that best matches the tone produced by tapping the corrugated drainage tube.

To everyone's surprise, the piano note matching the generated tone from the long corrugated tube is A_1^b (see figure 3), a semitone lower than the predicted A_1. In equal temperament, the frequency of the adjacent higher semitone is found by multiplying the starting frequency by $\sqrt[12]{2} = 1.059... = 1.06$, while the adjacent lower semitone is determined by multiplying by $\frac{1}{\sqrt[12]{2}} = 0.943... = 0.94$. Therefore, the observed frequency is approximately 6% lower than the predicted value. This observation is also true for the toy corrugated pipes, pointed out by Crawford [2] in 1974.

The discrepancy is analyzed with mathematical models in the following sections of this paper. For a general introductory class, the teacher can mention that the corrugations lower the speed of sound due to air pockets inside the rippled cavities. This fact is demonstrated in class by listening to the tone produced by a smooth 3 m (10 ft) pipe [1]. With the non-corrugated smooth pipe, the pitch is perceived to match the frequency estimated by the ideal open-pipe formula.

End corrections need not be mentioned since their effects apply to both the smooth and corrugated pipes. In discussing the lower pitch of the corrugated pipe relative to the higher pitch of the smooth pipe, the differential determining factor is the corrugated design of the drainage pipe.

End-pipe correction

Two factors come into play in extending the simple pipe model to more precise pipe models. These factors are the end-pipe and the corrugated-pipe corrections. The end correction is considered in this section, which applies to both the smooth pipe and the corrugated pipe.

The pressure nodes extend a little beyond each end of an open pipe, making the pipe length effectively longer. To correct for end effects,

$$\Delta L = 0.61r \tag{4}$$

must be added to the pipe length L for each open end, where r is the inner radius of the pipe and $kr \equiv \frac{2\pi r}{\lambda} << 1$ [7–9]. For the white smooth pipe used in the video [1], $r = 50.8\,\text{mm}$ (2.0 inches), and $\lambda = 2L = 2\,(3048\,\text{mm}) = 6096\ \text{mm}$ (20.0 feet). Therefore, $\frac{2\pi r}{\lambda} = \frac{2\pi(50.8\,\text{mm})}{6096\,\text{mm}} = 0.05 << 1$, which means that the end-correction equation (4) can be applied to the pipe. Equation (4) was derived by Levine and Schwinger [9] in 1948. Coauthor Julian Schwinger is the famous physicist who shared the 1965 Nobel Prize with Feynman and Tomonaga for work in quantum electrodynamics.

Incorporating the end correction, the pipe length is increased by adding equation (4) twice, once for each end, giving an effective length $L' = L + 2(0.61r) = L + 1.22r$. The revised formula for the fundamental frequency of equation (1) becomes

$$f = \frac{v}{2L'} = \frac{v}{2(L + 1.22r)}\,. \tag{5}$$

Since a finer model is being employed, a more precise value for the speed of sound at room temperature should be used as well as actual measurements of the dimensions of the specific pipe. These values are $v = 345\ \text{m}\cdot\text{s}^{-1}$ ($1132\ \text{ft}\cdot\text{s}^{-1}$), $L = 3.048\,\text{m}$ (10.0 ft), and $r = 50.8\,\text{mm}$ (2.0 inches). Using the above values in equation (5) gives the frequency

$$f = \frac{v}{2L'} = \frac{345\ \text{m}\cdot\text{s}^{-1}}{2(3.048 + 1.22\cdot 0.0508)\,\text{m}}$$

$$= 55.47 \text{ Hz} = 55 \text{ Hz}. \qquad (6)$$

Note that the uncorrected formula, equation (1), gives $f = \frac{v}{2L} = \frac{345 \text{ m} \cdot \text{s}^{-1}}{2(3.048 \text{ m})} = 56.59 \text{ Hz} = 57 \text{ Hz}$. Therefore, with this more detailed analysis, it can be seen that the end correction brings the fundamental more in line with the 55 Hz key on the piano. The discrepancy of 1.12 Hz between the corrected 55.47 Hz and uncorrected 56.59 Hz, before rounding, is 2%; i.e. the end correction lowers the pitch by 2%.

To better compare the theoretical corrected value of 55 Hz with experiment, a more precise measurement needs to be made for the fundamental produced by the smooth pipe than simply estimating its frequency on a keyboard. The sound from dropping the smooth white pipe in the video [1] can be analysed by the free audio software *Audacity* [10]. The sound of the dropped pipe produces the first few harmonics plotted in figure 4. The linear fit indicates that the fundamental frequency is 55 Hz, which is the value predicted by theory. Comparing the values before rounding off, the experimental value of 55.055 Hz from figure 4 is 0.412 Hz from the theoretical 55.467 Hz of equation (6), a discrepancy less than 1%.

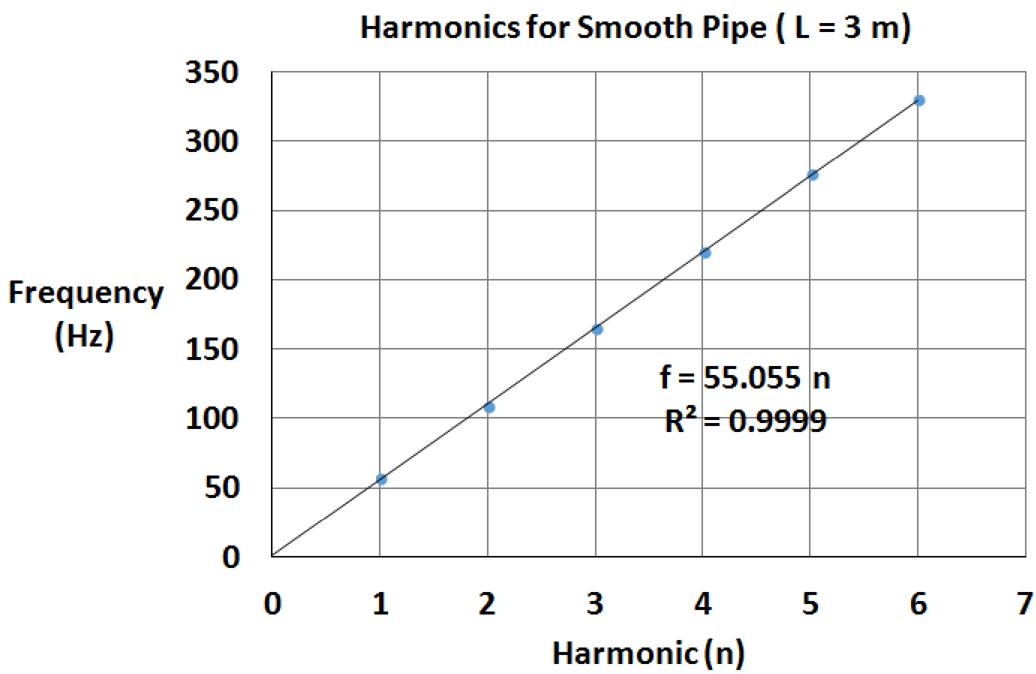

Figure 4. Plot of harmonics present in the spectrum when a smooth 3 m (10 ft) pipe is dropped. The linear fit gives a fundamental frequency of 55.055 = 55.1 Hz. The pitches are measured by the free audio software program *Audacity* [10].

The corrugation correction

The corrugated tube is more complicated because the end correction is accompanied by the additional corrugation correction. A further complication appears since one end of the corrugated tube is constructed with a small section having a slightly larger inner radius to allow the drainage tubes to be connected in series. The two inner radii of the corrugated pipe are $r = 52 \text{ mm}$ at one end and $R = 58 \text{ mm}$ at the larger end. Therefore, the effective pipe length, applying equation (4) appropriately for each end, is

$$L' = L + 0.61r + 0.61R = L + 0.61(r+R)\,. \quad (7)$$

One can think of the correction due to the corrugations in the pipe as a reduction in the speed of sound in the tube. The model of Nakiboğlu, Belfroid, Golliard and Hirschberg [11] can be employed to determine an estimate of the reduced sound speed. The relevant parameters in their model are found in figure 5, which is adapted from their paper [11].

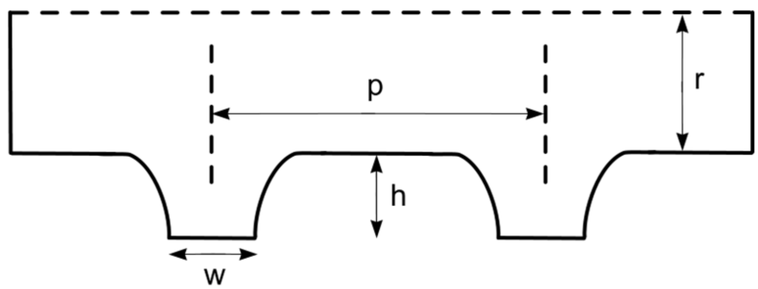

Figure 5. Corrugated-pipe section (not to scale) adapted from Nakiboğlu, Belfroid, Golliard and Hirschberg [11] with parameters relevant to the formula giving the speed of sound in the tube.

In figure 5, not drawn to scale, $r = 52$ mm, the inner radius of the pipe for the main length of the pipe. The parameter p is called the pitch, which gives the distance for one of the repeating corrugated patterns. The width of a corrugated ridge is w and the height of the ridge is h. The speed of sound in the pipe given in reference 11 is provided by Valdan, Gratton, Zendri and Oss [4] in the following very convenient form:

$$v' = v\frac{1}{\sqrt{1+\dfrac{2wh}{rp}}}\,. \quad (8)$$

The formula incorporating both the end and corrugated corrections is then $f' = \dfrac{v'}{2L'}$, where equation (7) is substituted for L' and equation (8) for v'. The combined result is

$$f' = \frac{v'}{2L'} = \frac{v}{\sqrt{1+\dfrac{2wh}{rp}}}\frac{1}{2[L+0.61(r+R)]}$$
$$= \frac{v}{2L}\frac{1}{\sqrt{1+\dfrac{2wh}{rp}}}\frac{1}{[1+0.61\dfrac{(r+R)}{L}]} \quad (9)$$

Since $\dfrac{2wh}{rp}$ and $0.61\dfrac{(r+R)}{L}$ are small quantities ε, we can use $(1+\varepsilon)^n \simeq 1+n\varepsilon$, with $n = -\dfrac{1}{2}$ for the velocity correction and with $n = -1$ for the end correction. With these excellent approximations, equation (9) can be replaced with $f' \simeq \dfrac{v}{2L}(1-\dfrac{wh}{rp})[1-0.61\dfrac{(r+R)}{L}]$, which can be further reduced to

$$f' \simeq \frac{v}{2L}\left[1-\frac{wh}{rp}-0.61\frac{(r+R)}{L}\right], \quad (10)$$

neglecting the very small second-order $2\frac{wh}{rp}[0.61\frac{(r+R)}{L}]$ term.

Equation (10) is very useful because it clearly reveals the corrections for the corrugation and pipe ends as subtracting from the uncorrected value. For the corrugated tube, $w = 8$ mm, $h = 7$ mm, $p = 18$ mm, $r = 52$ mm, $R = 58$ mm, and $L = 3083$ mm (10 ft and $1\frac{3}{8}$ inch by direct measurement). The parameters w and h of the corrugated ripples have low precision due to their small values and lack of uniformity in the corrugations throughout the pipe.

With the above values and $v = 345\ \mathrm{m \cdot s^{-1}}$, equation (10) becomes

$$f' \simeq 56.0(1 - 0.0598 - 0.0218) = 51.4\ \mathrm{Hz}. \quad (11)$$

Note that the reduction in pitch due to the end effects is 2% as found earlier with the smooth pipe. This 2% drop in pitch occurs for both the smooth and corrugated pipes. The additional drop in pitch due to the corrugations is predicted to be 6.0% from equation (11), which is essentially the 5.9% that corresponds to a semitone.

In order to directly compare the theoretical 51.4 Hz of equation (11) to the observed value, *Audacity* is used to obtain the best measured value in hertz for the corrugated fundamental. Figure 6 is a plot of the harmonics obtained from the sound produced by swinging the long corrugated drainage pipe over one's head as shown in the video [1]. The fit indicates an experimental fundamental equal to 52.0 Hz. Therefore, for the fundamental of the corrugated pipe, the experimental 52.0 Hz from the data in figure 6 is about 1% from the theoretical 51.4 Hz of equation (11).

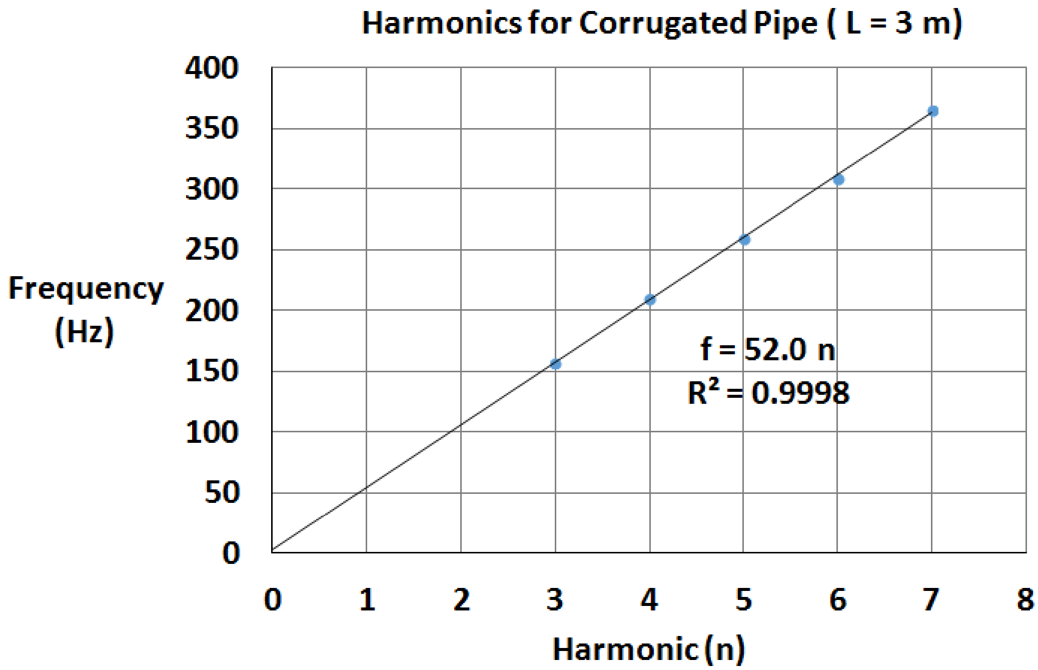

Figure 6. Plot of harmonics from twirling a corrugated 3 m (10 ft) pipe over one's head. The linear fit gives a fundamental frequency of 52.0 Hz. The pitches are measured by the free audio software program *Audacity* [10].

Additional fun demonstrations

The video [1] includes three related demonstrations. Two students can play the theme from the movie *Jaws* (1975), composed by John Williams, by tapping in an alternating fashion the 3 m (10 ft) corrugated pipe and a 3 m (10 ft) smooth pipe. This performance is possible since the *Jaws* theme repeatedly switches between two tones separated by a semitone, creating the characteristic suspenseful effect in the film. For another movie example, the toy version of the corrugated pipe can be used to play the beginning of the theme[2] from *2001: A Space Odyssey* (1968), which consists of harmonics 2, 3, 4, and 5 [5]. Note that the first harmonic is skipped when you twirl the toy tube [2]. Finally, one can take the monster version of the corrugated tube outside and twirl it with some amusing difficulty. Harmonics 3, 4, 5, 6, and 7 are briefly obtained in the video [1]. The demonstration of a professor or student attempting to twirl a 3 m (10 ft) flexible pipe overhead is very memorable.

Conclusion

A theatrical demonstration where students strike a 3 m (10 ft) corrugated drainage pipe and compare the experimentally generated tone to that predicted by basic theory persuasively introduces students to pipe physics. The actual pitch produced is a semitone (5.9%) lower in frequency as determined by matching the tone with a piano or Internet keyboard [6]. The contributing factor of the reduced pitch of the corrugated tube relative to a smooth pipe is due to the decrease in sound speed in the tube with the corrugations.

In a course for non-science majors one can present the mathematics of the ideal open pipe and stay away from the math involving the corrections. However, the teacher can briefly mention the cause for the pitch discrepancy in a conceptual manner, namely, the reduction in the speed of sound due to the corrugations. When a smooth 3 m (10 ft) pipe is dropped, the tone produced is perceived to be in agreement with the theoretical estimate form the ideal pipe model. For a higher-level course, the entire analysis found in this paper can be presented. Regardless of the student level, the drainage-pipe demonstration is visually striking, entertaining, and informative.

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