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The fourth harmonic and Sibelius

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Abstract

The composer Sibelius made ingenious use of artificial harmonics in his violin concerto. We reveal the underlying physics with an explanation suitable for the general student.

Vibrating strings present an excellent opportunity to bridge the gap too often present between the sciences and the arts, a gap pointed out by Snow decades ago [1]. A breathtaking 15 s in the violin concerto by Jean Sibelius illustrates an excellent blend of physics and music. Sibelius uses a physics principle to coax the string into its fourth harmonic pitch[1], producing exquisite high ethereal tones with a whistle-like quality. We developed a short video for students in which one of us (WM) demonstrates the effect with an explanation accessible to the layperson [2][2].

In an introductory physics course one learns that the natural modes of string vibrations, called the harmonics, form a series with the wavelength of the nth harmonic given by $\lambda_n = 2L/n$, where L is the length of the string. Diagrams of the first few harmonics, showing n half waves for the nth harmonic, can be found in introductory physics texts.

In figure 1 we illustrate the fourth harmonic, where a finger gently touches the string one-fourth of the way from the left end. A node is formed at this point and the string is thereby coaxed into vibrating at the fourth harmonic pitch. Note the presence of four half waves with their wavelength given by $\lambda_4 = 2L/4 = L/2$ (also evident by visual inspection of figure 1). The frequency produced is four times that of the first harmonic or fundamental. Since every time we double the frequency, we go up in pitch by an octave, the fourth harmonic pitch is two octaves higher than the fundamental pitch.
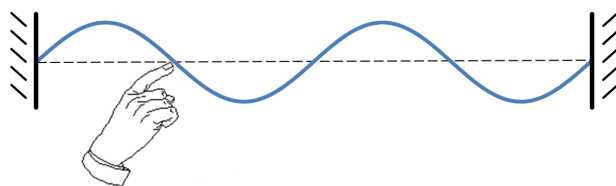

Figure 1. Coaxing the string to vibrate at the fourth harmonic.

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[1]We use the terminology 'fourth harmonic pitch' because in practice this sound has a Fourier spectrum with frequencies of the fourth, eighth and twelfth harmonics, and so on. Similarly, the fundamental pitch has a Fourier spectrum with frequencies of the first, second and third harmonics, and so on, characteristic of the usual violin timbre.

[2]One should be careful not to draw the conclusion that Sibelius used this trick to get pitches too high for the violin. One can reach the pitch obtained in figure 2 by firmly pressing at a distance much closer to the bridge. However, this would not have the desired timbre.

Figure 1 illustrates the physics model with the full length of the string. Violinists refer to the production of a harmonic using the entire length of a string as a natural harmonic. To play a melody on the string at the fourth harmonic pitch requires two fingers for each pitch. The violinist shortens the string by firmly pressing with one finger while gently touching with another finger. Violinists call this an artificial harmonic (see figure 2). The finger at the far left, as observed by the reader, is pressing firmly on the string, shortening the effective vibrating length of the string. The finger at the far right, as viewed by the reader, is gently touching the string to obtain a node at that point.

Figure 2. Violinist demonstrating an artificial harmonic. Photo credit: Christa Ruiz.

Since the node is produced one-fourth of the distance from the finger pressed firmly to the bridge, the fourth harmonic pitch is generated when the bow is applied. The resulting pitch is two octaves higher and perceived to be a 'whistle-like' ethereal sound. The distance between the two fingers gives the half-wavelength for the fourth harmonic.

We made a special video illustrating the artificial harmonics employed by Sibelius in the third movement of his violin concerto [2]. One of us (WM) performs the soloist part both with and without using artificial harmonics so that the listener can compare the melody of the fourth harmonic pitch (as composed by Sibelius) with what it sounds like as the fundamental pitch (using one finger only).

To further highlight the connection between physics and music, students can watch a violinist playing the relevant short section in the third movement of the Sibelius Violin Concerto within the context of the orchestration. An excellent orchestral performance on YouTube where one can see the fingers in action is that of soloist Maxim Vengerov with Daniel Barenboim conducting the Chicago Symphony[3][3]. The artificial harmonics begin at 30:46 and continue until 30:58. One should Start a little before at 30:30 or even earlier. I (MJR) like to first show my students our preparatory video [2], then ask them to raise their hands when the artificial harmonics begin to appear in the orchestral video [3].

In summary, we have presented an example that reveals the elegant physics behind a great masterpiece. Of course, the very act of playing the violin is based on the physics of strings. An artificial harmonic is especially rich in physics as it depends on the additional concept of coaxing a node at just the right place.

Acknowledgments

References

[1] Snow C P 1959 *The Two Cultures and the Scientific Revolution* (Cambridge: Cambridge University Press)

[2] William Michel demonstrates artificial harmonics. https://youtu.be/ILLg1FhRAKk

[3] The Sibelius Violin Concerto: soloist Maxim Vengerov with Daniel Barenboim conducting the Chicago Symphony Orchestra https://youtu.be/YsbrRAgv1b4

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[3]One should start at 30:30 and listen for the fourth harmonic sequence from 30:46 to 30:58.

Swiss-Spanish violinist **William Michel** is working on his master's degree in violin at the Jacobs School of Music, Indiana University, USA, under the tutelage of Mauricio Fuks. As a graduate of the Escuela Superior de Música Reina Sofía in Madrid, Spain, he performed extensively in Europe as a concert soloist, as a member of the quintet Todrà de Esteve, and in several orchestras.

Michael J Ruiz received his PhD in theoretical physics from the University of Maryland, USA in 1978. After graduating, he came to the University of North Carolina Asheville, USA and has taught there ever since. He is a classical and jazz pianist, integrating physics and music in his course *The Physics of Sound and Music*, which he has taught for over 30 years.

Coauthor Ruiz Comments. Another excellent example of the Sibelius where the fingers can be seen is violinist Sarah Chang at https://youtu.be/gpS_u5RvMpM with Jaap van Zweden conducting the Radio Filharmonisch Orkest (RFO). Advance to 29:49.The artificial harmonics section is from 30:03 to 30:14. You and your students might try to find other YouTube videos of the Sibelius.

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