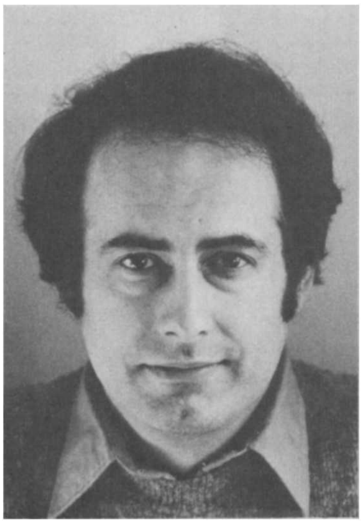
Modular synthesizers

MICHAEL J. RUIZ



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The music synthesizer is very useful in demonstrating a variety of principles of sound and has become quite popular in physics courses.[1-4] There exist many introductory sources on electronic music in both journals[1-5] and texts.[6-8] Also, numerous elementary discussions of synthesizers can be found in back issues of *Keyboard Magazine.*[9] This article will develop the basics of synthesizers from a modular perspective and with the aim of illustrating specific arrangements for the synthesis of musical sounds.

Although most of the effects are best demonstrated with a synthesizer, all the physics can be understood using just laboratory oscillators, filters, and an oscilloscope. Simple mathematical descriptions of the modified sound are given throughout in order to assist the reader without access to experimental equipment. A modular perspective is chosen in order to emphasize the underlying principles of all music synthesizers. For pedagogical purposes, the simplest sounds from the point of view of a physicist are described first.

Oscillators and musical range

The most fundamental module in a synthesizer is the oscillator. This unit produces periodic sound waves with frequencies falling within the range of human hearing. A typical frequency range for an audio oscillator found in a music synthesizer is 16 – 16 000 Hz. However, most applications employ frequencies below 4 kHz, a frequency corresponding approximately to the highest note on the piano.

The oscillator is usually controlled by a keyboard. When different keys are pressed, different voltages are sent to the oscillator. The voltage level determines the frequency of the oscillator. Since the oscillator frequency is controlled by voltage levels, the oscillator is referred to as a voltage-controlled oscillator, or VCO for short. The user can set the reference voltage that establishes the frequency for the lowest key on the keyboard. By adjusting this reference voltage, a three-octave keyboard (37 keys) can virtually span the entire range of human hearing. Voltage levels range from 0 – 5 V or 0 – 10 V.

A convenient notation that facilitates referring frequencies to musical ranges includes the corresponding note for the frequency on the piano. The middle C on the piano (note closest to the center of the piano where one can start playing the major scale using all white keys) is the fourth C found on the piano and is designated by C_4 (261.6 Hz). The lowest C is C_1 (32.7 Hz) and the highest note is C_8 (4186 Hz). Middle A, which is used by orchestras for tuning, is A_4 (440 Hz). The piano spans a little more than seven octaves due to three keys below C_1, each of which is labeled with a zero subscript; e.g., A_0 (27.5 Hz) is the lowest note on the piano. The piano serves as an excellent reference for musical range since the ranges of the traditional musical instruments lie within the large frequency range of the piano.

Table I

Waveforms and relative amplitudes for the first ten harmonics

Waveform	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Sine	1	0	0	0	0	0	0	0	0	0
Triangle	1	0	1/9	0	1/25	0	1/49	0	1/81	0
Ramp	1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10
Square	1	0	1/3	0	1/5	0	1/7	0	1/9	0
Pulse 33-1/3%	1	1/2	0	1/4	1/5	0	1/7	1/8	0	1/10
25%	1	$\sqrt{2}/2$	1/3	0	1/5	$\sqrt{2}/6$	1/7	0	1/9	$\sqrt{2}/10$
16-2/3%	1	$\sqrt{3}/2$	2/3	$\sqrt{3}/4$	1/5	0	1/7	$\sqrt{3}/8$	2/9	$\sqrt{3}/10$
~0%	1	1	1	1	1	1	1	1	1	1

Oscillator waveforms and characteristics

The typical synthesizer oscillator can produce a few different waveforms such as sine, triangle, ramp or sawtooth, and square waves. Each waveform provides for a different quality of sound or timbre. Most laboratory oscillators can produce one or two waveforms besides the usual sine wave, usually a square or triangle waveform.

A simple sine wave passing a fixed point in space can be expressed as

$$\psi(t) = A \sin(\omega t + \phi) \tag{1}$$

where A is the amplitude, ω is the angular frequency $2\pi f$ (f is the frequency), and ϕ is a phase term. The intensity, closely related to the loudness, varies as the square of the amplitude. The amplitude variation and corresponding changes in perceived loudness levels can be demonstrated with an oscillator, oscilloscope, amplifier, and speaker. The higher frequencies, perceived as higher pitches, appear on the oscilloscope display with shorter wavelengths. The wavelength of sound in air is related to frequency by the wave relation

$$\lambda f = v \tag{2}$$

where λ is the wavelength and v = 345 m/s, the speed of sound at room temperature.

A third important perceptual characteristic besides loudness and pitch is the quality of the sound, its timbre. This characteristic is easily demonstrated by switching quickly from a sine wave to a square wave using a laboratory oscillator. Immediately, the student can hear the richness of the square wave as the oscilloscope changes its display to the square-wave pattern.

The timbre of the sound is understood mathematically by Fourier's theorem, which states that any periodic wave of frequency f can be constructed from sine waves of frequencies f, $2f$, $3f$, $4f$, and so on. The sine wave with frequency f is called the fundamental and the other sine wave components ($2f$, $3f$, $4f$, etc.) are called overtones. The fundamental and overtones are also called harmonics, with the fundamental being the first harmonic.

A general periodic wave can be written as

$$F(t) = \sum_{n=1}^{\infty} A_n \sin(\omega_n t + \phi_n) \tag{3}$$

where $\omega_n = 2\pi n f$ and ϕ_n is the phase for the n^{th} harmonic. The function $F(t)$ represents any periodic waveform, e.g., a square or triangle wave, at a fixed point in space. The

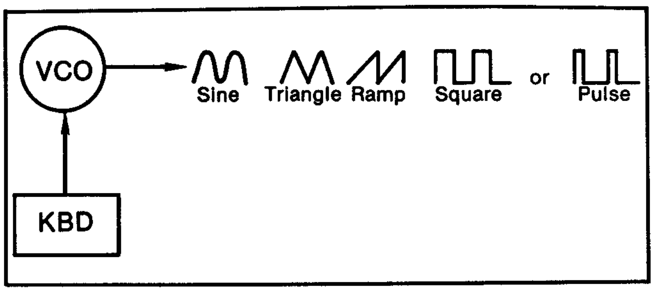

Fig. 1. Keyboard (KBD) controlling the voltage-controlled oscillator (VCO), which is capable of producing several different waveforms.

relative amplitudes for the first ten harmonics of several waveforms are given in Table I. The relative phases are not included since these are not important for synthesizer applications. Pulse waves are rectangular waveforms where the ratio of the pulse's width to the wavelength is given as a percentage (the duty cycle). Therefore, a square wave can be described as a pulse wave with a duty cycle of 50%. The simplest periodic waveform is obviously the sine wave. The more complex waves, e.g., the ramp and pulse waves, have rich Fourier spectra and consequently very full sounds.

Figure 1 illustrates the simple modular patch arrangement for a keyboard (KBD) and oscillator (VCO). When a key is pressed, a voltage is sent to the oscillator and the voltage remains at this value after the key is released. This feature is called sample and hold. The purpose of holding the voltage is to keep the frequency of the oscillator fixed, enabling other modules to shape and modify the sound from the oscillator. For example, by quickly turning an amplifier on and off electronically in different ways, the oscillator output can be made to sound like a plucking string or a woodwind instrument. These manipulations of the oscillator sound are discussed in later sections.

The oscillator in Fig. 1 is set to one of the available waveforms that the oscillator is capable of generating. The oscillator output must then be sent to an amplifier/speaker arrangement in order to be heard. Such final amplification is always necessary and will be understood throughout this article.

The notation employed in this article is basically that which is described in Ref. 10 and used extensively in Ref. 11. Sources of sound, e.g., audio oscillators and noise generators, are symbolized by circles. Modules like the keyboard that control other modules with voltage are called

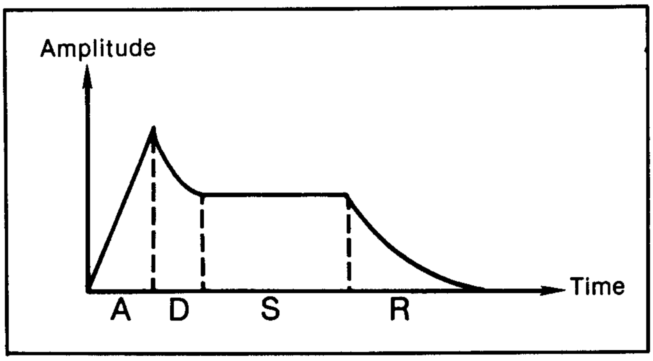

Fig. 2. The four phases of the envelope generator: attack (A), decay (D), sustain (S), and release (R).

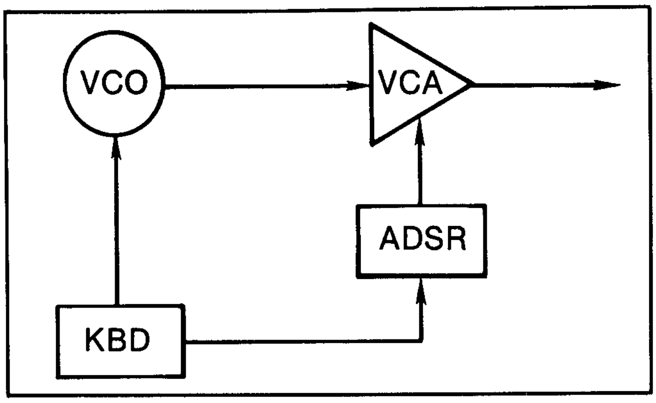

Fig. 3. The standard arrangement for the voltage-controlled oscillator (VCO), keyboard (KBD), envelope generator (ADSR), and voltage-controlled amplifier (VCA).

controllers and are represented by rectangles. Modules that accept audio signals for modification such as amplifiers and filters, are called modifiers. A modifier is designated by a triangle, the usual symbol for an amplifier or op amp. Audio signals flow from left to right and control voltages enter the modules they control from below.

Synthesizing simple musical sounds

The oscillator tone can be shaped by a special amplifier that can be turned on and off very fast by electrical means. In this way, a tone is produced that resembles the production of a musical sound. This special amplifier is controlled by voltage, and is called a voltage-controlled amplifier or VCA. The higher the input voltage, the more the volume is turned up. Common amplifiers for radios and televisions are adjusted by turning a volume control manually; the voltage-controlled amplifier is controlled electronically, allowing for changes in volume over time intervals of milliseconds.

Another module, the envelope generator, shapes the intensity or loudness of a waveform by controlling the voltage-controlled amplifier. The envelope generator controls four phases of the sound: attack, decay, sustain, and release. These phases are depicted in Fig. 2.

The envelope generator is activated by a trigger voltage, usually sent from the keyboard. When activated, the envelope generator produces an output voltage that begins to rise from zero to a maximum during a time interval set by the attack control. The attack time is typically somewhere between 1 ms and 2 s. Then, the decay phase begins, and lasts for a duration determined by the decay control (usually between 1 ms and 2 s). The sustain phase, which follows the decay, persists as long as a key from the keyboard is held down (trigger voltage still on). The sustain control fixes the voltage level that is maintained by the envelope generator during the sustain phase. When a key is released, the release phase is initiated with a time interval set by the release control (usually between 1 ms and 2 s).

Some envelope generators do not have a sustain phase. These are referred to as attack/release (AR) or attack/decay (AD) generators since the attack and release phases are then equivalent. There is no sustain phase in between to differentiate the two. An envelope generator with all four phases is abbreviated ADSR.

The shaping of a periodic waveform $F(t)$ by the envelope generator can be expressed as

$$\psi(t) = A_{EG}(t)F(t) \qquad (4)$$

where $A_{EG}(t)$ is the transient function of the envelope generator output, plotted in Fig. 2, and $F(t)$ is the steady-state or periodic waveform with a Fourier spectrum, described in Eq. (3). A demonstration of the effect of the transient function $A_{EG}(t)$ is easily performed by turning the volume quickly up and down, using a laboratory oscillator to supply the periodic waveform. Students can try this at home with their radios or television sets. However, the effect is not as dramatic as that achieved with the millisecond response time of an envelope generator controlling a voltage-controlled amplifier.

A block diagram of the standard modular arrangement or patch for the voltage-controlled oscillator (VCO), keyboard (KBD), envelope generator (ADSR), and voltage-controlled amplifier (VCA) is given in Fig. 3. The keyboard constantly sends a control voltage to the oscillator due to the sample and hold property of the keyboard. The oscillator output signal is then modified as it passes through the voltage-controlled amplifier. The volume of the amplifier is turned up and down by the control voltage of the envelope generator. The envelope generator begins its amplitude shaping of the oscillator output signal when it receives a trigger voltage from the keyboard, caused by the pressing of a key.

A variety of diverse sounds can be produced by choosing different oscillator waveforms and adjusting the envelope generator parameters. First approximations for several musical instruments can now be made. Table II lists the waveforms and attack/decay settings for approximating some common musical instruments. The decay/sustain controls are not very critical and have been omitted. Abrupt attacks or releases are very short times of about 1 or 2 ms; gradual attacks or releases are near 1 s. Careful adjustments are made experimentally. Knowledge of the physics of sound production and the Fourier spectra of periodic waves is helpful in the synthesis of musical sounds.

A simple flute or recorder consists of an open pipe. Standing waves in open pipes are sine waves, so the oscillator waveform is chosen accordingly. Gradual attack and release times are chosen since a flutist blows gently at the mouthpiece. The suggested playing range in Table II reflects a range that is typical for a flute.

Table II

First approximations for some musical instruments

Instrument	Waveform	Suggested Playing Range	Attack	Release
Simple Flute	Sine	440- 880 Hz (A_4-A_5)	Gradual	Gradual
Bass	Sine	65- 130 Hz (C_2-C_3)	Abrupt	Gradual
Clarinet	Triangle, Square	175- 700 Hz (F_3-F_5)	Gradual	Gradual
Violin	Ramp	880-1760 Hz (A_5-A_6)	Gradual	Abrupt
Harpsichord	Pulse (25%-33%)	440- 880 Hz (A_4-A_5)	Abrupt	Gradual
Banjo	Pulse (10%-25%)	220- 660 Hz (A_3-E_5)	Abrupt	Gradual

For a bass, the sound is generated by a vibrating string. Since standing waves on strings are sine waves, the sine wave is chosen for the first approximation of a bass. An abrupt attack and gradual release synthesize the familiar plucking sound of a bass. A lower playing range produces the long-wavelength tones characteristic of long bass strings.

A clarinet is a closed pipe due to the reed end, which is closed at the mouthpiece. Since the clarinet is more complex than a simple flute, the overtones present along with the fundamental cannot be neglected. In closed pipes, only standing waves with odd harmonics (f, $3f$, $5f$, $7f$, . . .) are formed. A glance at the Fourier spectra of Table I shows that the triangle and square waves have odd-harmonic Fourier spectra. Physics instructors often demonstrate how the square wave approximates the clarinet, using laboratory oscillators. By switching from the sine wave to the square, the student hears an abrupt change from a simple flute-like tone to the richer clarinet-like sound. If a triangle waveform is available, some may prefer it over the square wave in imitating a clarinet since the triangle waveform is not as harsh. An attack/release similar to the flute is chosen since the clarinet is likewise a woodwind instrument. Although any simple melody can be used to illustrate the synthesized clarinet-like sound, the opening clarinet line from *Rhapsody in Blue* by George Gershwin is very dramatic when synthesized.

The violin, although a string instrument, is usually played with a bow rather than plucked. A ramp wave approximates the vibrating shape obtained as a bow rubs against a string. When the bow leaves the string, the sound quickly dampens. A gradual attack and abrupt release simulate this effect.

The pulse wave is useful in synthesizing the complex plucking sounds of the harpsichord and banjo. The duty cycles given in Table II were estimated from an oscilloscope display. The values are approximate and to some extent subjective. Any reasonable attempt at playing a single line from a keyboard composition by Bach is quite convincing for a demonstration of the synthesized harpsichord. A rapid rendition of arpeggios (broken chords) improvising on a simple tune illustrates the banjo sound very well.

A filter can be inserted between the oscillator and amplifier in Fig. 3 in order to alter the Fourier spectrum of the complex wave from the oscillator. Such filtering of harmonics provides for an infinite variety of new tone qualities or timbres. The basic filter types are low-pass, high-pass, bandpass, and resonance. Low-pass (LP) filters pass harmonics below a certain cutoff frequency, while high-pass (HP) filters transmit harmonics with frequencies greater than the cutoff frequency. The cutoff frequency is fixed by an input control voltage. Bandpass (BP) filters allow harmonics within some intermediate frequency range to pass through. A resonance filter is a bandpass filter with a narrow transmission bandwidth and high gain for the central or resonant frequency. The central or resonant frequency is voltage-controlled and the ratio of the peak response to the bandwidth (the Q factor) can usually be adjusted.

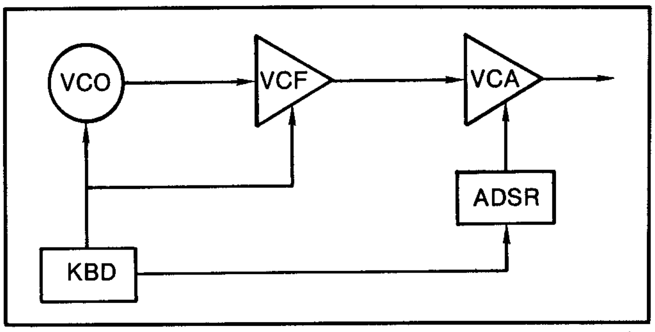

Fig. 4. The standard simple arrangement similar to Fig. 3, with the insertion of a voltage-controlled filter (VCF).

In order to provide for similar timbral shaping by a filter over a range of frequencies, it is necessary for certain filter characteristics such as the cutoff or central frequency to shift and adjust to the different incoming oscillator frequencies. If this shifting does not occur, as one plays up the scale using a low-pass filter, the tones will gradually disappear. The filter must track the keyboard, i.e., move its cutoff or central frequency to compensate for changing input audio frequencies, in order to shape the incoming waveforms as uniformly as possible. This tracking is accomplished by using the same voltage that controls the voltage-controlled oscillator to also control the filter. The synthesizer filter is often referred to as a voltage-controlled filter (VCF).

Figure 4 illustrates the standard use of the filter with voltage tracking supplied by the keyboard. However, depending on the baseline setting for the cutoff or central frequency relative to the playing range, the tracking may or may not provide for very uniform output waveforms as viewed on an oscilloscope. This is no cause for alarm since

Table III

Frequency modulation

Carrier		Modulator: Low ($\leqslant$ 1 Hz)	Modulator: Medium (2-5 Hz)	Modulator: High ($\sim$ 10 Hz)
	Low (5-20 Hz)	Starting Motor	Rough Idle	Motor Running
Carrier	Medium (300-600 Hz)	Siren	Police Siren	Science Fiction Gun
	High (1000-4000 Hz)	Toy Slide Whistle	Birds	Crickets

musical instruments do not preserve the same timbre over their playing ranges. The effects of filters on a complex waveform such as a square wave can be observed with a laboratory oscillator, filter, and oscilloscope. Mathematically, a periodic waveform described by Eq. (3) enters a filter with harmonic amplitudes A_n, and leaves the filter with a new set of harmonic amplitudes A_n' that depend on the filter characteristics. Also, the phases ϕ_n change.

Figure 4 shows the basic patch for the synthesis of musical sounds with no inharmonicities. Inharmonicities are spectral components that are not multiples of a fundamental and occur in nonperiodic sounds. Examples of inharmonic sounds are those made by bells, chimes, drums, and the piano. Although no musical sound is perfectly periodic, most musical sounds can be approximated by shaping periodic waveforms. However, the inharmonic sounds just mentioned cannot be approximated well at all with a single periodic waveform.

Drum sounds are the easiest of the inharmonic sounds to synthesize, using noise. Noise sources generate broadband noise, a little of all frequencies in a broad range of the audio spectrum (e.g., 100 – 10 000 Hz). Such noise is often referred to as white noise due to an analogy with light (an additive mixture of all the spectral colors produces white light). A good approximation to broadband noise can be obtained by turning the tuning dial on a radio so that reception lies where there are no stations broadcasting.

Noise is useful in synthesizing drum sounds, gunshots, and explosions. The noise is sent to the voltage-controlled amplifier. The envelope generator can control the amplifier with abrupt attacks and gradual releases. The results are explosive or percussive sounds. Sucking sounds are obtained with gradual attacks and abrupt releases. The keyboard may be employed to trigger the envelope generator; however, all keys give the same noise sound since the noise generator is not voltage-controlled like the oscillator.

Colorful noises (narrow-band noise) such as hisses can be produced with filters. If the keyboard tracking voltage controls the filter, noise entering the filter will change its quality depending on which key is pressed. A resonance filter emphasizes a central frequency bandwidth and is useful in synthesizing a group of whistlers. Any simple tune can be employed to demonstrate this whistling-effect; however, the theme from "The Bridge on the River Kwai" is a classic example.

Modulation and sweeping filter effects

A low-frequency control oscillator (1 – 25 Hz, low compared to audio frequencies) supplies a periodic control voltage that can be used to vary, i.e., modulate, sound characteristics by controlling modifiers. This low-frequency oscillator (LFO) is a controller and should be represented by a rectangle in modular diagrams so as not to be confused with the voltage-controlled oscillator.

Modifiers are designed so that they can receive more than one control voltage. When receiving two or more input control voltages simultaneously, the modifier responds to the algebraic sum of the voltages. A simple example is supplied by frequency modulation, where the voltage-controlled oscillator receives a fixed voltage from the keyboard and a periodic voltage from the control oscillator. The voltage-controlled oscillator responds by raising and lowering its pitch (frequency) accordingly as the control-oscillator voltage fluctuates. The keyboard fixes the carrier frequency (frequency without modulation) and the control oscillator is the modulator. The maximum amplitude of the control-oscillator voltage determines the sweeping range of the modulation, i.e., how much the carrier frequency is raised and lowered during the sweep. Such frequency sweeping is very easy to demonstrate using a laboratory oscillator. One simply turns the frequency control, sliding up to higher frequencies and returning back to lower frequencies.

When the control oscillator is set for low-voltage fluctuations, the result is a vibrato or quivering tone due to the small sweeping range. This is the common musical application for frequency modulation. However, a variety of everyday sounds such as sirens, starting motors, and singing birds can be synthesized with larger sweeping ranges. Table III gives approximate frequencies of the carrier and modulator in order to produce these sounds. The amplitude of the modulator voltage should be set to sweep about two or three octaves. For example, a police siren can be synthesized if a carrier frequency is chosen in the range of 300 – 600 Hz, with a modulator frequency around 2 – 5 Hz. If 500 Hz is taken for the carrier and 2 Hz for the modulator, then twice every second, the 500-Hz carrier will sweep up about two or three octaves (to 1000 or 1500 Hz). The modulating frequency for sirens is low enough to attempt making siren sounds by manually controlling a laboratory oscillator. A control oscillator producing a triangle modulating voltage creates more gradual sweeps than a sine wave and works better for the sounds in Table III. A square wave works fine for the carrier. The reported frequencies in Table III were estimated after the desired sounds were produced experimentally. They serve to give an order-of-magnitude understanding of the carrier and modulator frequencies.

Voltage-controlled oscillators often have special control inputs that permit pulse-width modulation, i.e., varying the duty cycle of a pulse wave. Rapid changes (around 10 Hz) in pulse width approximate the sound of many similar instruments playing together in unison, the "chorus effect." When similar instruments play together the same notes, the phases of the waveforms are always changing with respect to each other. The sweeping pulse width simulates the superposition of similar waveforms with different phases, creating a chorus effect.

The control oscillator can be employed to modulate filter characteristics by sweeping the cutoff or central frequency up and down. The result is a "wah-wah" sound. A 1-Hz sinusoidal control voltage, sweeping a filter with an input square wave audio signal, provides for excellent "wah-wah" effects. The "wah-wah" turns into a "wow-wow" for low audio frequencies. The Q value or resonance character of the filter is important. If the voltage-controlled oscillator can produce frequencies below 16 Hz, complex waveforms are heard as tapping sounds. With careful experimentation, sounds of dripping water can be synthesized by sending one of these extremely low-frequency complex waveforms through a sweeping low-pass filter.

Brass instruments can be approximated by sweeping the filter with the envelope generator. Figure 5 illustrates a simple patch for a brass instrument. A ramp wave is employed due to its raspy sound. A ramp wave contains both even and odd harmonics, which is very suitable, since the brass instruments behave as open pipes. The envelope generator must control both the filter and amplifier since in a simple synthesizer only one envelope generator is available. As the filter is quickly swept (a low-pass filter should be used), changes in timbre rapidly occur, imitating a brief note played on a brass instrument. Short attack and release times should be used. For this type of playing on a brass instrument, rapid bursts of air into the mouthpiece induce pressure variations and a changing Fourier spectrum.

Suggested keyboard ranges for the brass instruments are as follows: tuba (50 – 100 Hz), trombone (100 – 200 Hz), and trumpet (200 – 400 Hz). For a staccato passage, "In the Hall of the Mountain King" by Edvard Grieg is impressive when played on a synthesizer, especially in the tuba and trombone ranges. A more elaborate patch is required to obtain better brass imitations. The reader can find an excellent guide to improved brass sounds in Ref. 11. Here, a reverb (REV) module is added along with other modifications. Reverb creates reverberation and often gives sound a more natural texture and ambience, when used in moderation. However, lots of reverb is useful in synthesizing the echo of sounds in caves and dungeons. The reverb is a modifier module which is typically placed at the end of the audio chain.

The control oscillator can modulate the amplitude of a waveform by controlling the voltage-controlled amplifier. This effect is easily demonstrated with a radio by turning the volume up and down in a periodic manner. In musical terms, such amplitude modulation is referred to as tremolo. The mathematical expression Eq. (4) is an example of amplitude modification. It can readily be adapted to describe a simple periodic amplitude modulation by replacing $A_{EG}(t)$ with a sinusoidal control voltage $V_0 + V \sin \omega t$

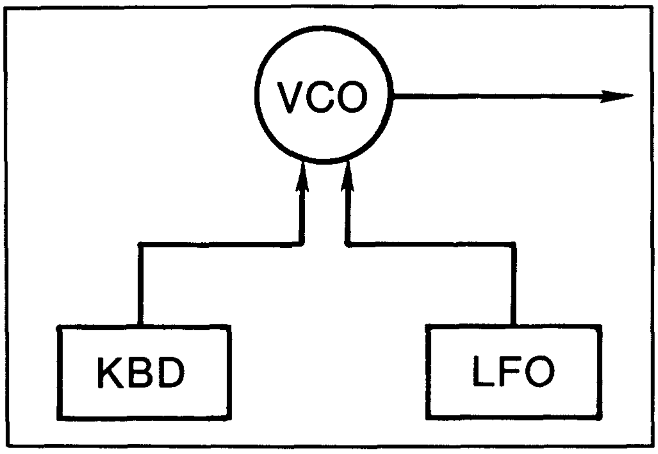

Fig. 5. Simple patch for synthesizing a brass instrument.

$$\psi(t) = (V_0 + V \sin \omega t)\, F(t) \tag{5}$$

where $F(t)$ is the carrier wave, described by Eq. (3).

When $V_0 = 0$, the modulation is balanced, i.e., both the carrier and modulator have the same (balanced) zero reference voltage. Balanced modulation is employed in a synthesizer module called the balanced or ring modulator in order to produce sounds with inharmonicities. It does this by creating the sum $(f_1 + f_2)$ and difference $(f_1 - f_2)$ tones for two incoming sine waves with frequencies f_1 and f_2 $(f_1 > f_2)$.

Such a result can be derived from Eq. (5) when $V_0 = 0$ and a sine wave is substituted for $F(t)$

$$\psi(t) = V \sin \omega_1 t \sin \omega_2 t \tag{6}$$

The trigonometric identity

$$\sin A \sin B = \frac{1}{2}\,[\cos(A - B) - \cos(A + B)] \tag{7}$$

can be used to show that Eq. (6) contains the sum and difference frequencies. Letting $A = \omega_1 t$ and $B = \omega_2 t$, the trigonometric identity Eq. (7) transforms Eq. (6) into

$$\psi(t) = \frac{V}{2}\,[\cos(\omega_1 - \omega_2)t - \cos(\omega_1 + \omega_2)t] \tag{8}$$

The wave described by Eq. (8) consists of two simple waves, one with frequency $f_1 - f_2$, and the other with frequency $f_1 + f_2$. Adding a relative phase between the two sine waves in Eq. (6) does not alter this result. Note that the modulator and carrier signals lose their identity and are interchangeable.

Very rich inharmonic sounds can be synthesized with a balanced modulator using complex waveforms. In order to analyze the spectrum of an inharmonic sound that is not too complicated, consider a sine wave ($f_1 = 500$ Hz) and a ramp wave ($f_2 = 50$ Hz) entering a balanced or ring modulator. The ramp wave must be broken down into its Fourier sine components in order to determine the spectrum for the synthesized inharmonic sound. The first three Fourier

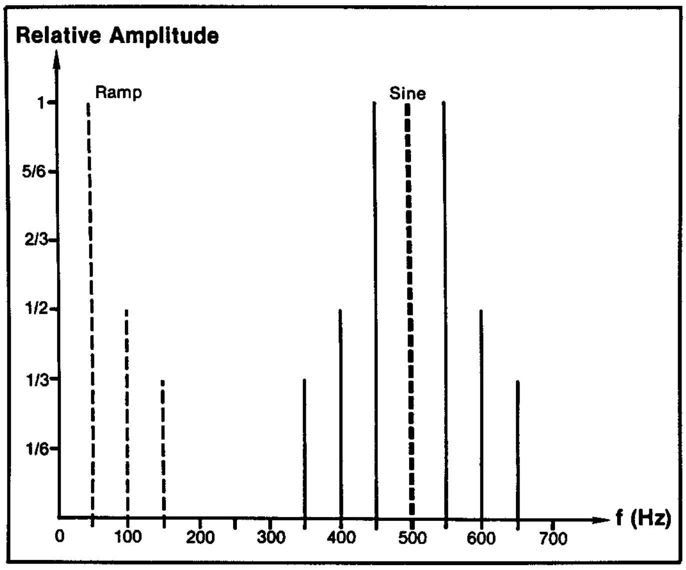

Fig. 6. Inharmonic spectrum formed by the first three harmonics of a 50-Hz ramp wave and a 500-Hz sine wave due to balanced modulation.

amplitudes for the ramp wave are given in Fig. 6 as dashed lines at 50 Hz, 100 Hz, and 150 Hz. The incoming sine wave with frequency f_1 = 500 Hz is also indicated in Fig. 6 by a dashed line. The ring modulator will produce sum and difference tones for each harmonic of the ramp wave combining with the sine wave at 500 Hz.

For the first harmonic of the ramp wave, the sum and difference tones are 500 Hz + 50 Hz = 550 Hz and 500 Hz − 50 Hz = 450 Hz. The second harmonic gives frequencies 400 Hz and 600 Hz, while the third harmonic produces simple waves with frequencies of 350 Hz and 650 Hz. These six simple waves are sine waves with appropriate phases. They are indicated in Fig. 6 with solid lines; however, the phases are not given. Note that the relative amplitudes of the ramp harmonics are preserved in the sum and difference tones. The spectral components below 500 Hz form the lower sideband; the spectral components above 500 Hz form the upper sideband. For simplicity, only the first three harmonics of the ramp wave were analyzed. The actual spectrum for the synthesized inharmonic sound includes the effects of the other harmonics as well. However, it can be seen from Table I that the relative amplitudes of the higher ramp harmonics get smaller and smaller. Therefore, their contributions to the overall inharmonic sound are not very great. The sine components of the balanced modulated sound given in Fig. 6 are not multiples of a fundamental like the spectral components for waveforms listed in Table I. The synthesized sound is characteristic of the inharmonic sounds produced by bells, chimes, and gongs.

Inharmonic sounds can be demonstrated without a ring modulator if two laboratory oscillators are available. Setting the oscillator frequencies so that they have no common fundamental will produce the eerie effect of inharmonicity. Complex waveforms provide for richer inharmonic sounds. By scanning frequencies with one oscillator, keeping the other oscillator frequency fixed, a variety of inharmonic sounds can be obtained.

Concluding remarks

A PAIA 4700/C Modular Synthesizer[12] was employed in developing the patch arrangements for this article. Modular synthesizers have no pre-wired connections and the modules are interfaced with patch cords for both audio and control voltage connections. Although somewhat inconvenient for performing musicians, the modular synthesizer is excellent for teaching purposes. The audio signal flow and control voltage paths are always transparent and given precisely by the modular block diagrams.

Modular synthesizers are economical for those who cannot purchase a complete synthesizer all at once. Modular kits can be obtained individually at prices less than $40 per module.[12] A voltage-controlled oscillator is recommended first. Then, an envelope generator along with a voltage-controlled amplifier can be added. Patch cords and audio connectors can be tailor-made from materials supplied by PAIA. Finally, a control oscillator/noise source and voltage-controlled filter complete the collection of basic synthesizer modules. An inexpensive regulated power supply and cabinet are available and recommended. A keyboard interface is somewhat expensive to add, with a price in the neighborhood of $200 for a three-octave keyboard. However, the envelope generator has a manual trigger which allows it to be controlled without a keyboard. In such an arrangement, a power supply voltage controls the frequency of the voltage-controlled oscillator.

Current prices for synthesizer module kits can be obtained by writing to PAIA.[12] However, experimenters should be experienced in soldering and basic electronic troubleshooting before attempting to build a modular synthesizer from a kit.

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