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Another Look at the Roche Limit

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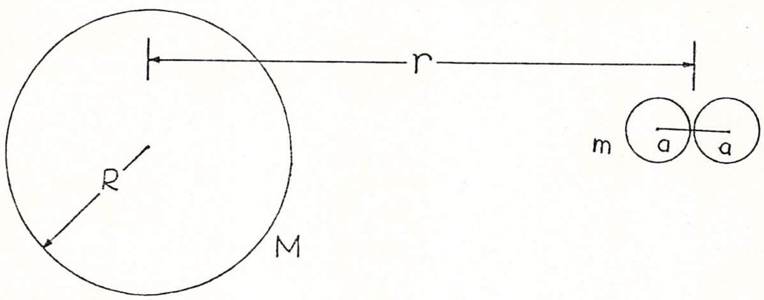
I enjoyed reading the recent article "The Roche Limit" by Karl Kochany in Phys 13 News.[1] It is a challenge to discuss the Roche limit in a quantitative manner at an introductory level.

I would like to share with you a noncalculus derivation of the Roche limit which Professor Morris S. Davis, an astronomer at the University of North Carolina (Chapel Hill, NC), showed me one afternoon after a seminar.

Consider two identical spherical masses, each with mass m and radius a, being pulled apart by a large spherical body of mass M and radius R.



The difference between the forces exerted by M on the centers of the little masses is

$$\Delta F = \frac{GMm}{(r-a)^2} - \frac{GMm}{(r+a)^2}, \qquad (1)$$

which simplifies to

$$\Delta F = \frac{4GMmar}{(r^2-a^2)^2} \approx \frac{4GMma}{r^3}, \qquad (2)$$

since r>>a.

The mutual attraction between the two small masses must be equal or less than this difference for the small masses to be torn apart.[1] Therefore,

$$\frac{Gmm}{(2a)^2} \leq \frac{4GMma}{r^3}, \qquad (3)$$

which can be written as

$$r \leq \left(\frac{16M}{m}\right)^{1/3} a. \qquad (4)$$

Using the density substitutions given in ref. 1, (4) becomes

$$r \leq \left(\frac{16\rho_p}{\rho_m}\right)^{1/3} R = 2.52 \left(\frac{\rho_p}{\rho_m}\right)^{1/3} R, \qquad (5)$$

where ρ_p = density of planet (large mass M) and ρ_m = density of moon (the moon here consists of the two small masses).

It is interesting to note that this simple noncalculus derivation produces a result that is extremely close to the result of Roche's original calculation, which gives[2]

$$r \leq 2.46 \left(\frac{\rho_p}{\rho_m}\right)^{1/3} R. \qquad (6)$$

This is somewhat fortunate since differential centripetal motion was neglected.[2] Students can readily recognize on a slide projection of Saturn that the rings lie within 2.5 Saturnian radii away from the planet's center.

[1]Karl Kochany, "The Roche Limit", Phys 13 News 49, pp. 6-7 (September 1982).

[2]Elske v. P. Smith and Kenneth C. Jacobs, Introductory Astronomy and Astrophysics (W.B. Saunders Company, 1 Goldthorne Avenue, Toronto, Ontario M8Z 5T9, 1973).

The Roche Limit and Saturn's Rings

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The paper by Karl Kochany[1] on the Roche limit raises some interesting points. So permit me to Roche the boat a little.

$$\text{He derives } r \leq \left(\frac{2\rho_p}{\rho_m}\right)^{1/3} R \qquad (1)$$

which includes the constant 2.

I. The value of this constant seems to depend on the method used to derive it. Here is another derivation.

The acceleration due to the gravity of the planet, at distance r from its centre, is

$$a = \frac{GM}{r^2}$$