Physics I with Calculus, Prof. Ruiz (Doc), UNC-Asheville (1978-2021), DoctorPhys on YouTube
Chapter F. Friction. Prerequisite: Calculus I. Corequisite: Calculus II.

F0. Friction. We first introduce the friction formulas. Then we spend most of the chapter doing problems. What happens when you try to push something to slide it across the floor?

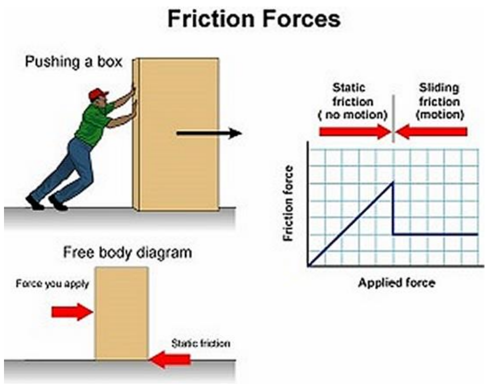

Figure Courtesy Vishakha.malhan, Wikimedia
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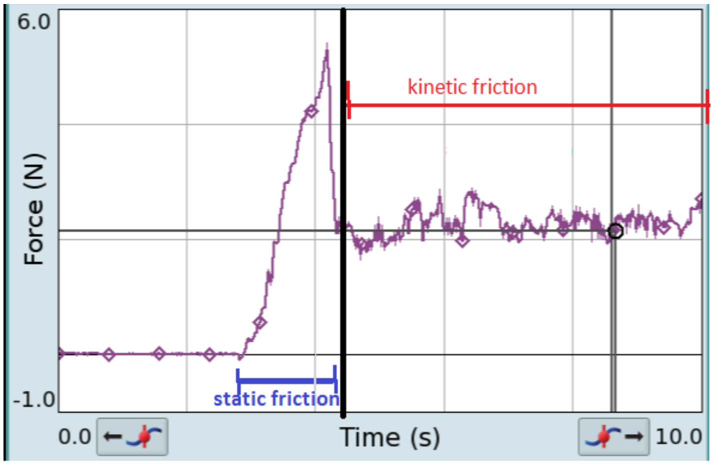
In the figure a person is pushing a tall rectangular box. As force is applied, nothing happens at first. As the applied force increases, still nothing happens up to a point. Then the box budges and starts moving. At this transition point, the force necessary to keep the box moving is usually less than the force needed to make it budge. Check out that force graph in the figure where the friction force is plotted against applied force in an ideal model. Therefore, we need to define two types of friction: static and dynamic (kinetic). The convention is use the word kinetic for the moving case.

To a good approximation, the maximum static frictional force and the kinetic frictional force are proportional to the normal force that the ground surface pushes up on the mass. In this case, the normal force is simply the weight. We introduce constants of proportionality for the static and kinetic cases and note that the static frictional force builds up to a point as you push. In all cases friction opposes you.

Static friction formula: $f_s \le \mu_s N$.

Kinetic friction formula: $f_k = \mu_k N$.



Data "Collected in physics lab using a Vernier Dual-Range Force sensor."

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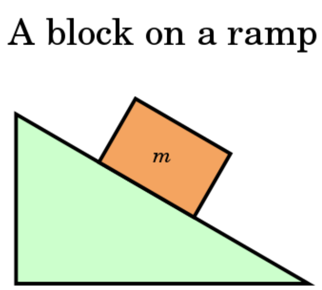
Compare the real-life data at the left with the idealized model in the figure at the top of the page.

Approximate values of some coefficients of friction are given below in the table. Note that the static coefficient is usually less than the kinetic coefficient. Also, note that the coefficient can exceed 1 in some cases.

Approximate Values of Some Coefficients of Friction

Material Combinations		Surface Conditions	Static μ_s	Kinetic μ_k
Aluminum	Aluminum	Clean and Dry	1.05 – 1.35	1.4
Aluminum	Mild Steel	Clean and Dry	0.61	0.47
Brass	Steel	Clean and Dry	0.51	0.44
Copper	Cast Iron	Clean and Dry	1.05	0.29
Glass	Glass	Clean and Dry	0.9 – 1.0	0.4
Glass	Glass	Greasy	0.1 - 0.6	0.09 – 0.12
Ice	Ice	Clean at 0° C	0.1	0.02
Leather	Cast Iron	Clean and Dry	0.6	0.56
Nickel	Nickel	Clean and Dry	0.7 – 1.1	0.53
Nickel	Nickel	Greasy	0.28	0.12
Rubber	Dry Asphalt	Clean and Dry	0.9	0.5 - 0.8
Wood - Waxed	Wet Snow	Clean	0.14	0.1

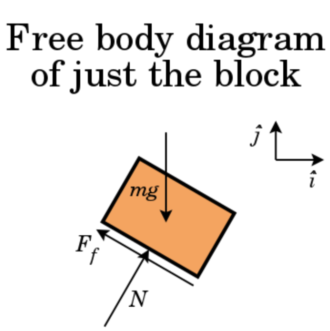
Data Courtesy Engineering ToolBox (2004). Friction – Friction Coefficients and Calculator. Website Link (Accessed January 27, 2022)



“Block on a ramp (top) and corresponding free body diagram of just the block (bottom).” Courtesy Krishnavedala, Wikimedia. Creative Commons Pubic Domain Dedication License

The figure at the left shows a block on a ramp. Then, below it we find the free body force diagram giving the three forces on the block.

1. Weight. The force is $W = mg$ and it points down.

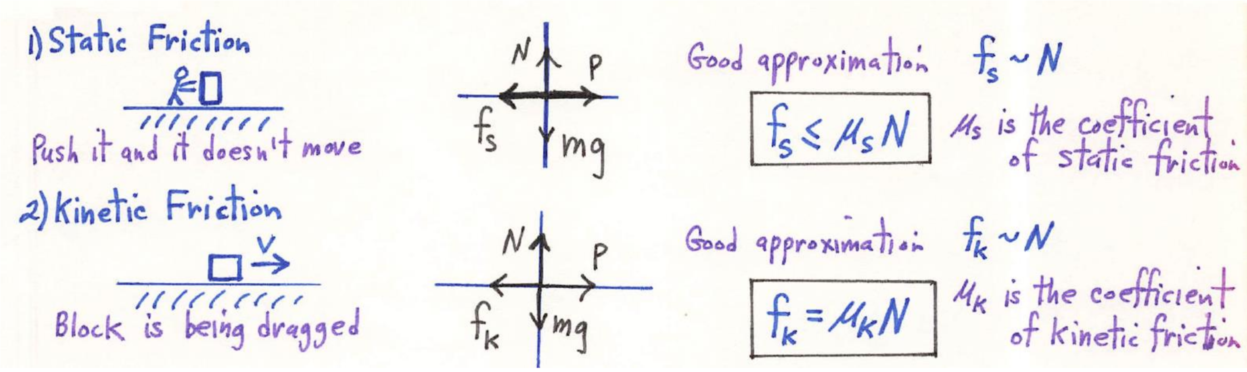


2. Normal. The normal force N pushing perpendicular to the block.

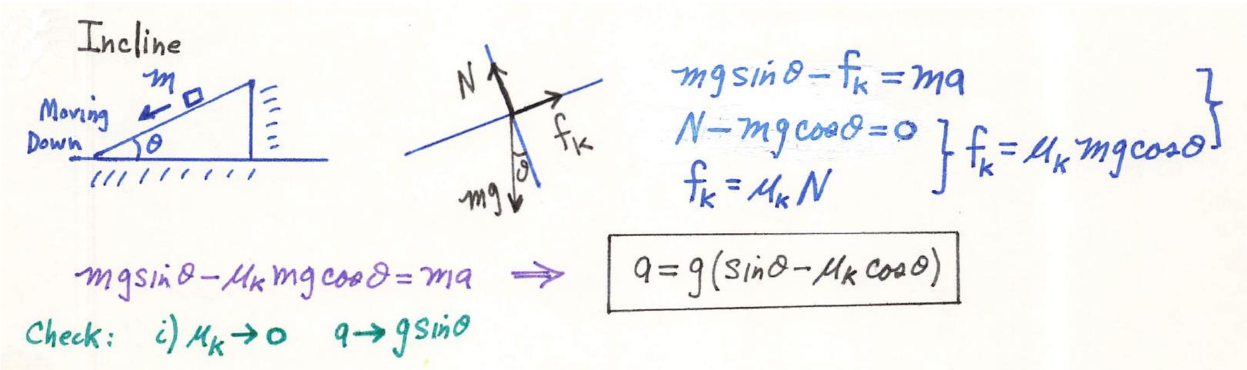
3. Friction. The force of static friction $F_f \equiv f_s \le \mu_s N$ along the bottom of the block, opposing the block’s tendency to slide down.

We will proceed to work ten problems where friction is included. Our friction formulas are auxiliary formulas that we will use in our force equations.

Below are two more examples of friction. Then we start our problem marathon.



F1. Incline and Friction. We return to the incline but now include friction. In our problem below, a mass is sliding down an incline. Find the acceleration a.



Above you see (i) the sketch, (ii) the force diagram, (iii) the equations, and (iv) the solution. We have added an important auxiliary equation to step (iii). This equation is the friction equation. The three equations are again listed below. I chose "+" down the incline and "+" along the normal. I indicate these choices below. However, it is best to label these choices in your diagrams too.

$$\sum F_{\text{down incline}} = mg\sin\theta - f_k = ma$$

$$\sum F_{\text{normal to incline}} = N - mg\cos\theta = 0$$

Friction Equation: $f_k = \mu_k N$

Looking at these three equations, we can establish a technique in solving. My method is to start with the last equation. I remember my method as bottom up. I start with the last equation and work my way upwards. I begin with $f_k = \mu_k N$ and then substitute in for the normal considering the next equation above the frictional one: $N - mg\cos\theta = 0$. With $N = mg\cos\theta$ inserted into $f_k = \mu_k N$, we get

$$f_k = \mu_k mg\cos\theta .$$

The last step is to substitute our equation $f_k = \mu_k mg\cos\theta$ into the first of the three equations, i.e., $mg\sin\theta - f_k = ma$.

$$mg\sin\theta - f_k = ma$$

$$mg\sin\theta - \mu_k mg\cos\theta = ma$$

Now we can complete the solution for the acceleration a. The mass factor divides out.

$$g\sin\theta - \mu_k g\cos\theta = a$$

$$(\sin\theta - \mu_k\cos\theta)g = a$$

$$\boxed{a = (\sin\theta - \mu_k\cos\theta)g}$$

The friction decreases the acceleration. The dimensions check out since the trig functions and coefficient of friction μ_k are dimensionless. We can see that the answer is reasonable by considering the answer for no friction.

$$\lim_{\mu_k \to 0} a = \lim_{\mu_k \to 0}(\sin\theta - \mu_k\cos\theta)g = g\sin\theta$$

This result we derived in the last chapter.

What about angle extremes?

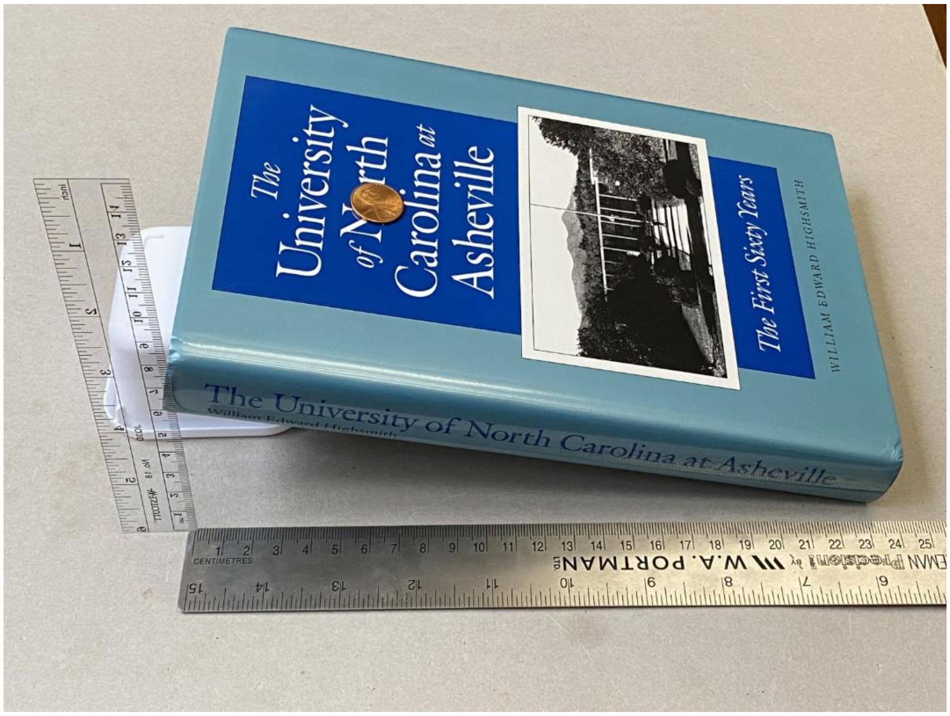
$$\lim_{\theta \to 90^\circ} a = \lim_{\theta \to 90^\circ}(\sin\theta - \mu_k\cos\theta)g = (\sin 90^\circ - \mu_k\cos 90^\circ)g = (1 - \mu_k \cdot 0)g = g$$

We are in free fall. For the other angle extreme, where the incline is flat, we must be careful. The normal force will be equal to the weight but we no longer have motion. And since there is no push or pull sideways on the mass, there is no frictional force. So we are left with

$$a = (\sin\theta - \mu_k\cos\theta)g \underset{\mu_k = 0}{\rightarrow} g\sin\theta \underset{\theta = 0^\circ}{\rightarrow} g\sin 0^\circ = 0\,.$$

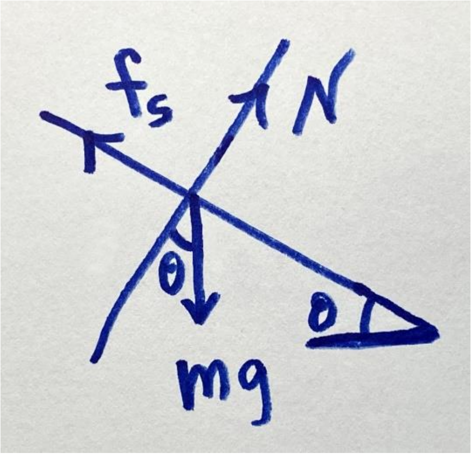
The mass just sits there.

F2. Measuring the Coefficient of Static Friction. There is a simple classic experiment you can do to measure coefficients of static friction. Take a coin and put it on a book. Then lift one side of the book to increase the angle making an incline steeper and steeper until the coin starts sliding.



Here is the basic setup using a book about my college, The *University of North Carolina at Asheville*, written by our first Chancellor, William Highsmith.

Watch! You only need the angle. For coin on book with glossy cover we can arrive at the coefficient of static friction. You keep increasing the angle until the coin budges.



The equations are

$$\sum F_{\text{down incline}} = mg\sin\theta - f_s = ma = 0$$

$$\sum F_{\text{normal to incline}} = N - mg\cos\theta = 0$$

Friction Equation: $f_s = \mu_s N$ (when coin budges)

The maxed out condition is met when the coin budges and at this threshold, i.e., right before it, there is still no motion ($a = 0$). The last two equations give

$$f_s = \mu_s N = \mu_s mg\cos\theta\,.$$

Using this equation in $mg\sin\theta - f_s = 0$, leads to $mg\sin\theta - \mu_s mg\cos\theta = 0$.

$$mg\sin\theta = \mu_s mg\cos\theta$$

The mg divides out.

$$\sin\theta = \mu_s \cos\theta \quad \Rightarrow \quad \tan\theta = \mu_s$$

The coefficient of static friction is given by the tangent.

$$\boxed{\mu_s = \tan\theta}$$

This result is true on any planet!

The specific setup in our photo with the coin and book was actually very close for the coin to begin slipping. It is best to do this experiment with a lab partner so that the length measurements are more accurate rather than my placing the rulers somewhat haphazardly. But you get the idea. Take the following approximate values from the rulers in the photo for the calculation: a rise (y) of 6 cm and a run of 23 cm (x). The slope is then rise/run, the tangent.

$$\mu_s = \tan\theta = \frac{\text{height}}{\text{length}} = \frac{6\text{ cm}}{23\text{ cm}} = \frac{6}{23} = 0.26$$

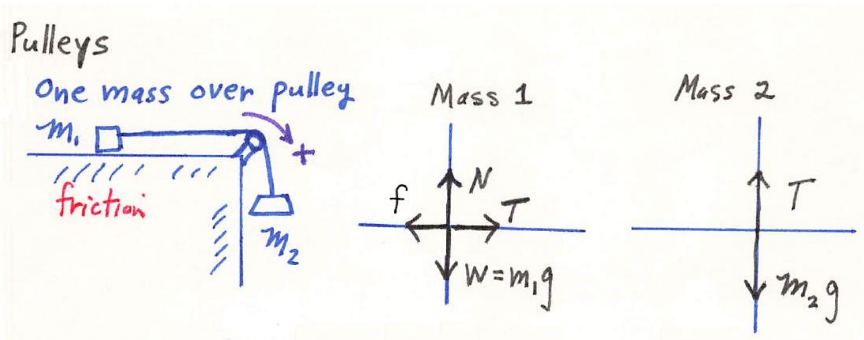
We didn't even need to calculate the angle. But we can.

$$\theta = \tan^{-1}\frac{6}{23} = 14.6° = 15°$$

If you are making this measurement, you should repeat the experiment several times and take an average. If your surfaces are not uniform, you will find variations, depending on where you place the coin on the book.

F3. Mass on Table with Pulley. Let's revisit our mass m_1 on the table being pulled by a second mass m_2, but this time we add friction between mass m_1 and the table top.

The sketch, force diagrams, and three equations plus the friction equation are given below.



$$T - f = m_1 a$$

$$N - m_1 g = 0$$

$$m_2 g - T = m_2 a$$

$$f = \mu_k N$$

Using our bottom-up method, we start with the last equation $f = \mu_k N$. Now, we need to skip up to the $N - m_1 g = 0$ to get the normal force that we need in the friction equation. Use the normal equation $N = m_1 g$ in $f = \mu_k N$.

$$f = \mu_k N = \mu_k m_1 g$$

It is always most elegant to get things in terms of the parameters originally given in the problem: m_1, m_2, and μ_k. At this point, we can move up to the first equation $T - f = m_1 a$.

Then, $T - f = m_1 a$ with $f = \mu_k m_1 g$ becomes

$$T - \mu_k m_1 g = m_1 a .$$

Add the above equation to $m_2 g - T = m_2 a$,
remembering the Newton's Third Law Trick: Action and Reaction.

$$T - \mu_k m_1 g = m_1 a$$
$$m_2 g - T = m_2 a$$

The action and reaction forces cancel. The results are below.

$$m_2 g - \mu_k m_1 g = m_1 a + m_2 a$$

$$(m_2 - \mu_k m_1) g = (m_1 + m_2) a$$

$$\boxed{a = (\frac{m_2 - \mu_k m_1}{m_1 + m_2}) g}$$

Dimensions look good and we recover our formula from before when $\mu_k \to 0$.

$$a = \lim_{\mu_k \to 0} (\frac{m_2 - \mu_k m_1}{m_1 + m_2})g = (\frac{m_2}{m_1 + m_2})g$$

This result is what we found in the last chapter in Section E2!

For the tension, use $T - \mu_k m_1 g = m_1 a$ and substitute in for the acceleration $a = (\frac{m_2 - \mu_k m_1}{m_1 + m_2})g$.

$$T = m_1 a + \mu_k m_1 g$$

$$T = m_1 (\frac{m_2 - \mu_k m_1}{m_1 + m_2})g + \mu_k m_1 g$$

$$T = (\frac{m_1 m_2 - \mu_k m_1^2}{m_1 + m_2})g + \mu_k m_1 g$$

$$T = (\frac{m_1 m_2}{m_1 + m_2})g - (\frac{\mu_k m_1^2}{m_1 + m_2})g + \mu_k m_1 g$$

$$T = (\frac{m_1 m_2}{m_1 + m_2})g + \left[\frac{-\mu_k m_1^2}{m_1 + m_2} + \frac{(m_1 + m_2)}{(m_1 + m_2)} \mu_k m_1 \right] g$$

$$T = (\frac{m_1 m_2}{m_1 + m_2})g + \left[\frac{-\mu_k m_1^2 + \mu_k m_1^2 + m_2 \mu_k m_1}{m_1 + m_2} \right] g$$

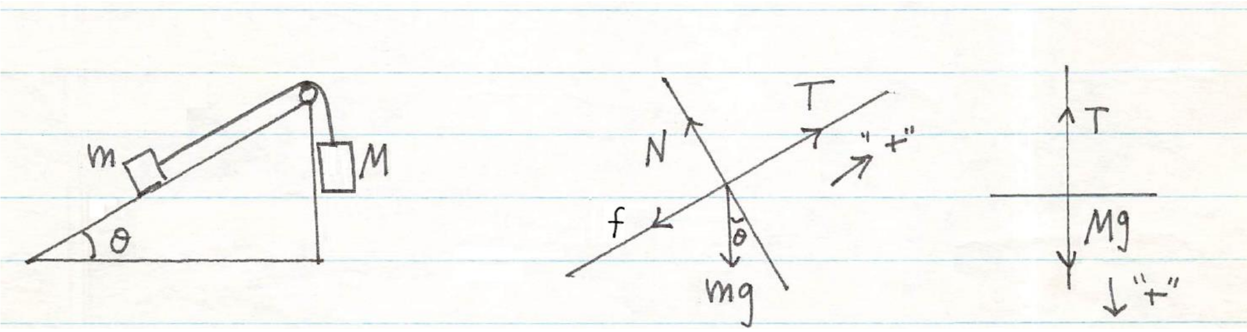
$$T = (\frac{m_1 m_2}{m_1 + m_2})g + (\frac{\mu_k m_1 m_2}{m_1 + m_2})g$$

$$\boxed{T = (\frac{m_1 m_2}{m_1 + m_2})(1 + \mu_k)g}$$

Dimensions look good and we recover our formula from Section E2 when $\mu_k \to 0$.

$$\lim_{\mu_k \to 0} T = \lim_{\mu_k \to 0} (\frac{m_1 m_2}{m_1 + m_2})(1 + \mu_k)g = (\frac{m_1 m_2}{m_1 + m_2})g$$

F4. Pulling a Mass up an Incline. We did this one before with no friction. Assume motion with M >> m. Find a formula that will insure that M pulls m up the incline. The coefficient of kinetic friction is μ_k.



There are two equations for m, one for M, and the auxiliary equation for the friction.

$$T - f - mg\sin\theta = ma$$

$$N - mg\cos\theta = 0$$

$$Mg - T = Ma$$

$$f = \mu_k N$$

Again, my favorite way is to work from the bottom and the normal force.

$$f = \mu_k N = \mu_k mg\cos\theta$$

I have no idea of the equation I am trying to derive.
But I know I do NOT want to see f, N, or T in the final equation for a.
I want to see parameters given in the problem: m, M, μ_k, θ, and of course g.

So far, we have

$$T - f - mg\sin\theta = ma,$$

$$Mg - T = Ma,$$

$$f = \mu_k mg\cos\theta.$$

Add the first two equations to get rid of T.

$$T - f - mg\sin\theta + Mg - T = ma + Ma$$

$$-f - mg\sin\theta + Mg = ma + Ma$$

Now use $f = \mu_k mg\cos\theta$.

$$-\mu_k mg\cos\theta - mg\sin\theta + Mg = ma + Ma$$

$$-(\mu_k\cos\theta + \sin\theta)mg + Mg = (m + M)a$$

$$\frac{-(\mu_k\cos\theta + \sin\theta)mg + Mg}{m + M} = a$$

$$a = \frac{Mg - (\mu_k\cos\theta + \sin\theta)mg}{m + M}$$

$$a = \frac{[M - m(\mu_k\cos\theta + \sin\theta)]}{m + M}g$$

$$\boxed{a = \frac{[M - m(\sin\theta + \mu_k\cos\theta)]}{M + m}g}$$

We want $a > 0$. Therefore,

$$M - m(\sin\theta + \mu_k\cos\theta) > 0.$$

$$\boxed{M > m(\sin\theta + \mu_k\cos\theta)}$$

We can find the tension T easily from $Mg - T = Ma$

$$T = Mg - Ma$$

with $a = \frac{[M - m(\sin\theta + \mu_k\cos\theta)]}{M + m}g$.

$$T = Mg - M\frac{[M - m(\sin\theta + \mu_k\cos\theta)]}{M + m}g$$

$$T = M\left\{1 - \frac{[M - m(\sin\theta + \mu_k\cos\theta)]}{M + m}\right\}g$$

$$T = M\left\{\frac{M+m}{M+m} - \frac{\left[M - m(\sin\theta + \mu_k \cos\theta)\right]}{M+m}\right\}g$$

$$T = M\left\{\frac{M+m-\left[M - m(\sin\theta + \mu_k \cos\theta)\right]}{M+m}\right\}g$$

$$T = M\left\{\frac{M+m-M+m(\sin\theta + \mu_k \cos\theta)}{M+m}\right\}g$$

$$T = M\left\{\frac{m+m(\sin\theta + \mu_k \cos\theta)}{M+m}\right\}g$$

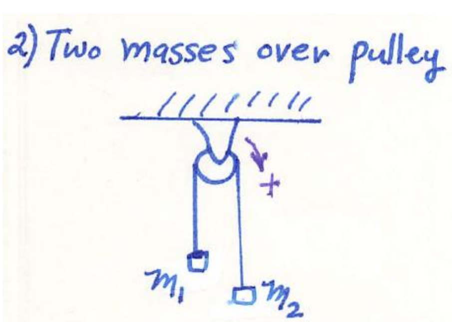
$$T = M\left\{\frac{\left[m(1+\sin\theta + \mu_k \cos\theta)\right]}{M+m}\right\}g$$

$$\boxed{T = \left(\frac{Mm}{M+m}\right)(1+\sin\theta + \mu_k \cos\theta)g}$$

Summary.

$$\boxed{a = \frac{[M - m(\sin\theta + \mu_k \cos\theta)]}{M+m}g} \qquad \boxed{T = \left(\frac{Mm}{M+m}\right)(1+\sin\theta + \mu_k \cos\theta)g}$$

Remember the two masses (no friction) from the last chapter?



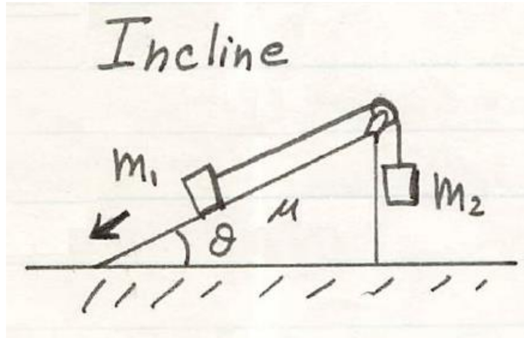
$$\boxed{a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g}$$

$$\boxed{T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g}$$

Verify that these check out from the above master equations when $\theta = 90°$.

Why does the coefficient of friction go away in this limit?

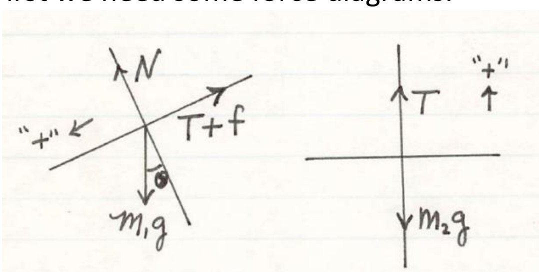
F5. Sliding Down an Incline, Hauling a Mass Up.



Find the general formula for the acceleration and tension in the cable for mass m_1 sliding down the incline and pulling up m_2. Give the condition that makes this motion possible.

Wait, didn't we just do this problem? No, because with friction you have to be super careful. The friction now will point in the opposite direction

First we need some force diagrams.



(i) Sketch. See the above hand-drawn figure.

(ii) Force Diagram. See left.

(iii) Force Equations. The equations for each mass are given under each force diagram.

$$m_1 g \sin\theta - T - f = m_1 a \qquad T - m_2 g = m_2 a$$
$$N - m_1 g \cos\theta = 0$$
$$f = \mu N$$

(iv) Solution. Start at the bottom with $f = \mu N$. Then move up to $N - m_1 g \cos\theta = 0$, finding $N = m_1 g \cos\theta$. These two equations lead to

$$f = \mu N = \mu m_1 g \cos\theta .$$

Now go to the first equation and substitute in for f.

$$m_1 g \sin\theta - T - f = m_1 a$$

$$m_1 g \sin\theta - T - \mu m_1 g \cos\theta = m_1 a$$

$$m_1 g (\sin\theta - \mu \cos\theta) - T = m_1 a$$

The next step is to add the equation for mass 2, where the tension T is equal and opposite. You could have done this first and then substitute in for the friction.

$$m_1 g (\sin\theta - \mu \cos\theta) - T = m_1 a \qquad T - m_2 g = m_2 a$$

The addition brings us to

$$m_1 g(\sin\theta - \mu\cos\theta) - T + T - m_2 g = m_1 a + m_2 a \,.$$

$$m_1 g(\sin\theta - \mu\cos\theta) - m_2 g = m_1 a + m_2 a$$

We want the acceleration on one side of the equation.

$$m_1 g(\sin\theta - \mu\cos\theta) - m_2 g = (m_1 + m_2)a$$

$$a = \frac{m_1 g(\sin\theta - \mu\cos\theta) - m_2 g}{(m_1 + m_2)}$$

$$\boxed{a = \frac{\left[m_1(\sin\theta - \mu\cos\theta) - m_2\right]g}{m_1 + m_2}}$$

The simplest T equation to get the tension is $T - m_2 g = m_2 a$.

$$T = m_2 a + m_2 g$$

$$T = m_2 \frac{\left[m_1(\sin\theta - \mu\cos\theta) - m_2\right]g}{(m_1 + m_2)} + m_2 g$$

$$T = m_2 \frac{\left[m_1(\sin\theta - \mu\cos\theta) - m_2\right]g}{(m_1 + m_2)} + m_2 \frac{(m_1 + m_2)}{(m_1 + m_2)} g$$

$$T = \frac{\left[m_1 m_2(\sin\theta - \mu\cos\theta) - m_2^2\right]g}{(m_1 + m_2)} + \frac{(m_1 m_2 + m_2^2)}{(m_1 + m_2)} g$$

$$T = \frac{\left[m_1 m_2(\sin\theta - \mu\cos\theta) - m_2^2\right]g + (m_1 m_2 + m_2^2)g}{(m_1 + m_2)}$$

$$T = \frac{\left[m_1 m_2(\sin\theta - \mu\cos\theta)\right]g + (m_1 m_2)g}{(m_1 + m_2)}$$

$$\boxed{T = \frac{m_1 m_2(1 + \sin\theta - \mu\cos\theta)}{m_1 + m_2} g}$$

Summary (with $\mu = \mu_k$). $a = \frac{\left[m_1(\sin\theta - \mu_k \cos\theta) - m_2\right] g}{m_1 + m_2}$ $T = \frac{m_1 m_2 (1 + \sin\theta - \mu_k \cos\theta)}{m_1 + m_2} g$

The dimensions look good and we can do a quick check when $\theta = 90°$, the two hanging masses. We found in the last chapter: $a = (\frac{m_2 - m_1}{m_1 + m_2})g$ and $T = (\frac{2m_1 m_2}{m_1 + m_2})g$, where the plus direction was mass 2 moving down. The problem we just did is set up for mass 1 moving down. So we should get

$$a = (\frac{m_1 - m_2}{m_1 + m_2})g \qquad T = (\frac{2m_1 m_2}{m_1 + m_2})g$$

Let's see.

$$\lim_{\theta \to 90°} a = \frac{\left[m_1(\sin 90° - \mu_k \cos 90°) - m_2\right] g}{m_1 + m_2} = \frac{\left[m_1(1 - \mu_k \cdot 0) - m_2\right] g}{m_1 + m_2} = \frac{m_1 - m_2}{m_1 + m_2} g$$

$$\lim_{\theta \to 90°} T = \frac{m_1 m_2 (1 + \sin 90° - \mu_k \cos 90°)}{m_1 + m_2} g = \frac{m_1 m_2 (1 + 1 - \mu_k \cdot 0)}{m_1 + m_2} g = \frac{2m_1 m_2}{m_1 + m_2} g$$

Can we get the solution for sliding down from the solution sliding up? Is there a shortcut?

Sliding Up: $a = \frac{[M - m(\sin\theta + \mu_k \cos\theta)]}{M + m} g$ $T = (\frac{Mm}{M + m})(1 + \sin\theta + \mu_k \cos\theta) g$

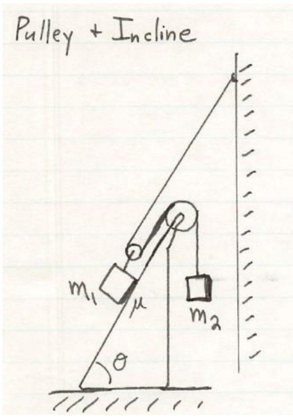
To relate the masses in each problem: $m = m_1$ and $M = m_2$

Sliding Up: $a = \frac{[m_2 - m_1(\sin\theta + \mu_k \cos\theta)]}{m_2 + m_1} g$ $T = (\frac{m_2 m_1}{m_2 + m_1})(1 + \sin\theta + \mu_k \cos\theta) g$

Sliding Down: $a = \frac{\left[m_1(\sin\theta - \mu_k \cos\theta) - m_2\right] g}{m_1 + m_2}$ $T = \frac{m_1 m_2 (1 + \sin\theta - \mu_k \cos\theta)}{m_1 + m_2} g$

Why does letting $a \to -a$ and $\mu_k \to -\mu_k$ do the trick?

F6. Incline and Two Pulleys. Consider the incline and pulley arrangement below. We propose a problem with some values in this case to balance our rather theoretical problems above.

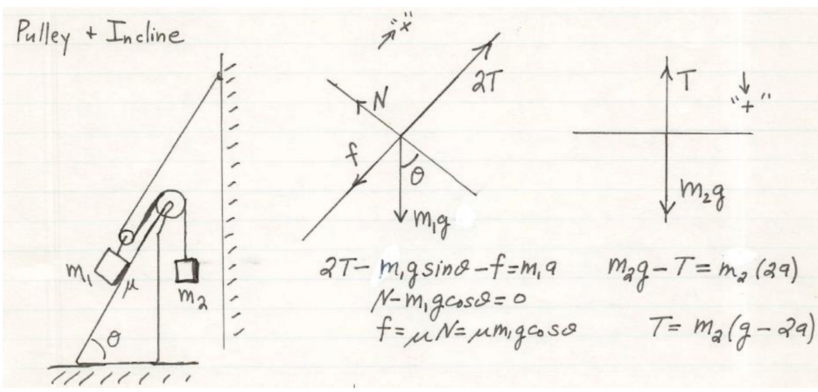


(1) What is the acceleration in terms of g for the conditions where $m_1 g = 25$ N, $m_2 g = 75$ N, $\theta = 70.0°$, and $\mu_k \equiv \mu = 0.20$?

(2) What is the acceleration formula for the case $\theta = 90.0°$? What is the condition to balance the two masses when $\theta = 90.0°$?

(3) What is the acceleration in terms of g if $m_1 g = 82$ N, $m_2 g = 22$ N, $\theta = 60.0°$, and $\mu = 0.21$?

Solution. We are going to need an equation for the acceleration.



Let's choose up the incline as the positive direction. **Note that if we are wrong, we will have to redo things since the frictional force will have to be flipped!** Always remember that friction is not simple like our frictionless problems we did before, when a negative sign just means we are going the other way and we flip the sign on our numerical answer. In this case, if we are wrong, we will have to recalculate the numerical answer! Next there is another subtlety in this particular problem – the doubling of the rope pull!

Note that when a rope is doubled up over a pulley that the two ropes work together. Everywhere in the rope the tension is T. **Therefore, the pull up the incline here is 2T.** This principle of pulleys is powerful as you can double your strength this way, and with more pulleys even more.

But there is a price to pay. When mass m_1 moves up the incline a distance d, mass m_2 moves twice this distance. You do not get anything for free. Mass m_2 has to pull through twice the distance. This feature is also evident from the fact that the rope at m_1 is doubled up. The equations are listed below.

$$2T - m_1 g \sin\theta - f = m_1 a$$

$$N - m_1 g \cos\theta = 0$$

$$f = \mu N = \mu m_1 g \cos\theta$$

$$m_2 g - T = m_2(2a)$$

Substitute $f = \mu m_1 g \cos\theta$ into the first equation $2T - m_1 g \sin\theta - f = m_1 a$.

$$2T - m_1 g \sin\theta - \mu m_1 g \cos\theta = m_1 a$$

At this stage we normally add the other equation with the tension, which is $m_2 g - T = m_2(2a)$.

But the first equation has $2T$ so we first multiply the second one by 2.

$$2m_2 g - 2T = 2m_2(2a)$$

Now we can add the two following equations.

$$2T - m_1 g \sin\theta - \mu m_1 g \cos\theta = m_1 a$$
$$2m_2 g - 2T = 2m_2(2a)$$

The result is $2T - m_1 g \sin\theta - \mu m_1 g \cos\theta + 2m_2 g - 2T = m_1 a + 2m_2(2a)$.

$$-m_1 g \sin\theta - \mu m_1 g \cos\theta + 2m_2 g = m_1 a + 2m_2(2a)$$

$$-m_1 g \sin\theta - \mu m_1 g \cos\theta + 2m_2 g = (m_1 + 4m_2)a$$

$$2m_2 g - m_1 g \sin\theta - \mu m_1 g \cos\theta = (m_1 + 4m_2)a$$

$$(m_1 + 4m_2)a = 2m_2 g - m_1 g \sin\theta - \mu m_1 g \cos\theta$$

$$(m_1 + 4m_2)a = 2m_2 g - m_1 g(\sin\theta + \mu \cos\theta)$$

$$a = \frac{2m_2 g - m_1 g(\sin\theta + \mu\cos\theta)}{m_1 + 4m_2}$$

$$\boxed{a = \left[\frac{2m_2 - m_1(\sin\theta + \mu\cos\theta)}{m_1 + 4m_2}\right] g}$$

(1) What is the acceleration in terms of g for the conditions where $m_1 g = 25\text{ N}$, $m_2 g = 75\text{ N}$, $\theta = 70.0°$, and $\mu_k \equiv \mu = 0.20$?

$$a = \frac{2m_2 g - m_1 g(\sin\theta + \mu\cos\theta)}{m_1 + 4m_2} = \frac{2(75\text{ N}) - (25\text{ N})(\sin 70° + 0.2\cos 70°)}{(25\text{ N} + 4\cdot 75\text{ N})/g}$$

$$a = \left[\frac{2(75\text{ N}) - (25\text{ N})(\sin 70° + 0.2\cos 70°)}{(25\text{ N} + 4\cdot 75\text{ N})}\right] g$$

$$a = \left[\frac{124.798\text{ N}}{325\text{ N})}\right] g$$

$$\boxed{a = 0.38 g}$$

(2) What is the acceleration formula for the case $\theta = 90.0°$? What is the condition for you to be able to balance the two masses?

$$\boxed{a = \left[\frac{2m_2 - m_1(\sin\theta + \mu\cos\theta)}{m_1 + 4m_2}\right] g}$$

$$a = \left[\frac{2m_2 - m_1(\sin 90° + \mu\cos 90°)}{m_1 + 4m_2}\right] g$$

$$a = \left[\frac{2m_2 - m_1(1 + \mu\cdot 0)}{m_1 + 4m_2}\right] g$$

$$\boxed{a = (\frac{2m_2 - m_1}{m_1 + 4m_2}) g}$$

In order to balance the masses, $a = 0$.

$$\boxed{m_1 = 2m_2}$$

This result is expected due to doubling of the rope for mass m_1.

Mass m_2 can support twice its mass, i.e., $m_1 = 2m_2$.

(3) What is the acceleration in terms of g if $m_1 g = 82\text{ N}$, $m_2 g = 22\text{ N}$, $\theta = 60.0°$, $\mu = 0.21$?

$$a = \frac{2m_2 g - m_1 g(\sin\theta + \mu\cos\theta)}{m_1 + 4m_2}$$

$$a = \frac{2m_2 g - m_1 g(\sin\theta + \mu\cos\theta)}{m_1 g + 4m_2 g} g$$

$$a = \frac{2(22\text{ N}) - (82\text{ N})(\sin 60° + 0.21\cos 60°)}{(82\text{ N} + 4\cdot 22\text{ N})/g}$$

$$a = (\frac{44 - 79.624}{170})g \quad => \quad a = -0.21g$$ **BAD**

Because now we learn that the friction was set up the wrong way.

I need to flip the sign on the friction term to fix it.

$$a = \frac{2m_2 g - m_1 g(\sin\theta + \mu\cos\theta)}{m_1 + 4m_2} \underset{\mu \to -\mu}{\longrightarrow} \frac{2m_2 g - m_1 g(\sin\theta - \mu\cos\theta)}{m_1 + 4m_2}$$

$$a = \frac{2m_2 g - m_1 g(\sin\theta - \mu\cos\theta)}{m_1 + 4m_2}$$

$$a = \frac{2m_2 g - m_1 g(\sin\theta - \mu\cos\theta)}{m_1 g + 4m_2 g} g$$

$$a = \frac{2(22\text{ N}) - (82\text{ N})(\sin 60° - 0.21\cos 60°)}{(82\text{ N} + 4\cdot 22\text{ N})/g}$$

$$a = (\frac{44 - 82\cdot 0.761025}{170})g$$

$$a = (\frac{44 - 82\cdot 0.761025}{170})g \quad => \quad a = -0.11g$$ **GOOD NOW!**

If you do not like this minus sign, set up the problem with "+" going the other way.

Then, "+" will mean going down the incline and your acceleration will be $a = +0.11g$.

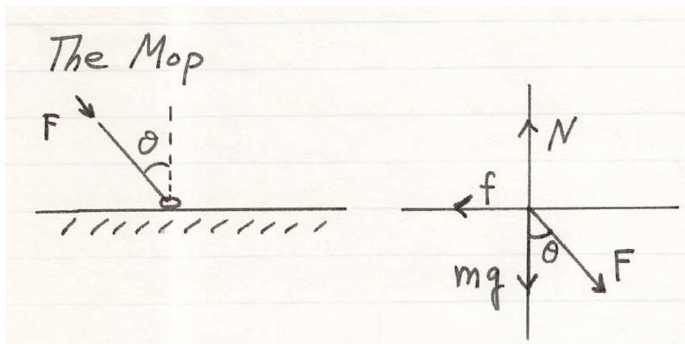
In that case the general formula will become

$$a = \frac{-2m_2 g + m_1 g(\sin\theta - \mu\cos\theta)}{m_1 + 4m_2}\,.$$

F7. The Mop. A force is applied along the handle of a mop. The mop has mass m and you can neglect the mass of the handle. The coefficient of static friction between the mop and the floor is μ_s and the coefficient of kinetic friction between the mop and the floor is μ_k.

(1) Find the force necessary so that the mop moves across the floor at a constant speed.
(2) Find the angle such than no matter what force you apply, the mop will not move.

Courtesy Jim Champion, Flickr
License: Attribution-ShareAlike 2.0 Generic (CC BY-SA 2.0)
Swabbing the Floors. March 13, 2010.



A sketch (i) and free body diagram (ii) appear above. There are 4 forces acting on the mop. Newton's Laws (iii) come next. Let positive "x" be to the right and positive "y" up.

$$\sum F_x = F\sin\theta - f = ma = 0$$

$$\sum F_y = N - F\cos\theta - mg = 0$$

Note that the acceleration along the x-direction is zero since the mop moves with constant speed across the floor. These formulas are given below along with the "auxiliary equation" for the friction.

$$F\sin\theta - f = 0$$

$$N - F\cos\theta - mg = 0$$

$$f = \mu N$$

The coefficient of friction when the mop is moving is μ_k.

The coefficient of friction when the mop is at rest and about to budge is μ_s.

Now it is time for step (iv), to solve the equations and answer the questions.

(1) Force for constant speed. The equations are

$$F \sin\theta - f_k = 0,$$

$$N - F\cos\theta - mg = 0,$$

$$f = \mu_k N\ .$$

As mentioned before, I like to work up from the last equation.

$$f = \mu_k N \qquad N = F\cos\theta + mg$$

$$f = \mu_k \left(mg + F\cos\theta\right)$$

Use this equation in the first equation $F \sin\theta - f_k = 0$.

$$F \sin\theta - f_k = 0$$

$$F \sin\theta = \mu_k \left(mg + F\cos\theta\right)$$

Solve for F .

$$F \sin\theta - \mu_k mg - \mu_k F\cos\theta = 0$$

$$F(\sin\theta - \mu_k \cos\theta) - \mu_k mg = 0$$

$$F(\sin\theta - \mu_k \cos\theta) = \mu_k mg$$

$$\boxed{F = \frac{\mu_k mg}{\sin\theta - \mu_k \cos\theta}}$$

(2) What is the necessary angle such than no matter what force you apply, the mop will not move. Now we need the coefficient of static friction. The same equation though applies, but with the coefficient of static friction.

$$F = \frac{\mu_s mg}{\sin\theta - \mu_s \cos\theta}$$

We want $F \to \infty$ so that we would need an infinite force.

This condition is met when the denominator goes to zero.

$$\sin\theta - \mu_s \cos\theta = 0$$

$$\sin\theta = \mu_s \cos\theta$$

$$\tan\theta = \mu_s$$

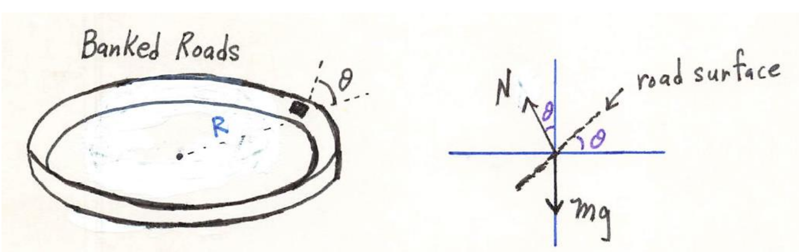
$$\boxed{\theta = \tan^{-1}\mu_s}$$

F8. Banked Roads.

"A steeply banked ramp from eastbound California State Route 92 to northbound Interstate 880 in Hayward, California. The ramp was banked in order to fit a cloverstack interchange into the existing footprint of what was formerly a cloverleaf interchange. Photographed by Wikimedia user Coolcaesar on July 19, 2020." Courtesy Coolcaesar, Wikimedia. License: [Attribution-ShareAlike 4.0 International (CC BY-SA 4.0)].

Consider a road with black ice to get a sense of what no friction entails. A banked road is needed to keep the car on the road. We first work with this no-friction case.

1. No Friction. Consider a banked road with radius R, designed for speed v_0 when there is no friction, e.g., super icy conditions. Find an equation for θ in terms of v_0, R, and g.



(i) Sketch. Far left.

(ii) Force Diagram.

(iii) Newton's Laws

Pick "x" towards center as "+": $\sum F_x = N\sin\theta = m\frac{v_0^2}{R}$

Pick "y" up as "+": $\sum F_y = N\cos\theta - mg = 0$

DO NOT pick axes along the slant and perpendicular to the tilted road! Why Not?
Because the circle for the acceleration lies along the horizontal.
So we pick West-East as "x" and North-South as "y".

(iv) Solve.

$$N\sin\theta = m\frac{v_0^2}{R}$$

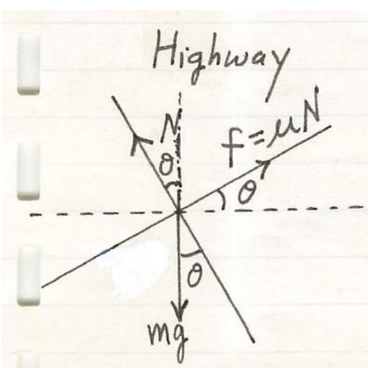
$$N\cos\theta = mg$$

Divide the equations: $\boxed{\tan\theta = \frac{v_0^2}{gR}}$.

2. Friction. Consider a banked highway with the following design characteristics:

Designed for $v_0 = 64.0\,\frac{\text{km}}{\text{h}}$ (40 miles/h) if there were no friction (i.e., super icy),
with radius of curvature $R = 200.0\text{ m}$ (660 ft).

On a rainy day, what is the minimum μ_s needed
to prevent sideways slipping down at $v = 40.0\,\frac{\text{km}}{\text{h}}$ (25 miles/h) .



Solution.

(i) Sketch. A car at the origin is traveling on a banked highway.

(ii) Force Diagram. For speeds $v < v_0$ the frictional force is up the slanted road because now the angle is too steep due to the slower speed.

(iii) Newton's Laws. The acceleration is towards the center of a horizontal circle. Therefore,

$$N\sin\theta - f\cos\theta = m\frac{v^2}{R}$$

$$N\cos\theta + f\sin\theta = mg$$

$$f = \mu_s N$$

Start with the last equation as we like to do and substitute the friction equation $f = \mu_s N$ into the other equations.

$$N \sin\theta - \mu_s N \cos\theta = m\frac{v^2}{R}$$

$$N \cos\theta + \mu_s N \sin\theta = mg$$

Factoring out the normal force, the pair of equations becomes

$$N(\sin\theta - \mu_s \cos\theta) = m\frac{v^2}{R} ,$$

$$N(\cos\theta + \mu_s \sin\theta) = mg \ .$$

Divide them.

$$\frac{\sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta} = \frac{v^2}{gR}$$

The normal forces cancel and so do the mass appearances in the numerator and denominator. Solve for μ_s.

$$\frac{\sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta} = \frac{v^2}{gR}$$

$$\sin\theta - \mu_s \cos\theta = \frac{v^2}{gR}(\cos\theta + \mu_s \sin\theta)$$

$$\sin\theta - \mu_s \cos\theta = \frac{v^2}{gR}\cos\theta + \frac{v^2}{gR}\mu_s \sin\theta$$

$$\sin\theta = \frac{v^2}{gR}\cos\theta + \frac{v^2}{gR}\mu_s \sin\theta + \mu_s \cos\theta$$

$$\sin\theta - \frac{v^2}{gR}\cos\theta = \frac{v^2}{gR}\mu_s \sin\theta + \mu_s \cos\theta$$

$$\sin\theta - \frac{v^2}{gR}\cos\theta = \mu_s(\frac{v^2}{gR}\sin\theta + \cos\theta)$$

$$\mu_s(\frac{v^2}{gR}\sin\theta+\cos\theta)=\sin\theta-\frac{v^2}{gR}\cos\theta$$

$$\mu_s(\frac{v^2}{gR}\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\cos\theta})=\frac{\sin\theta}{\cos\theta}-\frac{v^2}{gR}\frac{\cos\theta}{\cos\theta}$$

$$\mu_s(\frac{v^2}{gR}\tan\theta+1)=\tan\theta-\frac{v^2}{gR}$$

$$\mu_s(1+\frac{v^2}{gR}\tan\theta)=\tan\theta-\frac{v^2}{gR}$$

$$\boxed{\mu_s=\frac{\tan\theta-\dfrac{v^2}{gR}}{1+\dfrac{v^2}{gR}\tan\theta}}$$

That's a cool equation. Since the banked road was designed to support v_0 with no friction, the numerator must be zero for v_0 so we get $\mu_s=0$ as expected. Therefore, $\tan\theta=\frac{v_0^2}{gR}$, a result we already found in our no-friction banked problem earlier. Then

$$\mu_s=\frac{\tan\theta-\dfrac{v^2}{gR}}{1+\dfrac{v^2}{gR}\tan\theta}$$

can be written as

$$\mu_s=\frac{\dfrac{v_0^2}{gR}-\dfrac{v^2}{gR}}{1+\dfrac{v^2}{gR}\dfrac{v_0^2}{gR}}$$

after substituting $\frac{v_0^2}{gR}$ for $\tan\theta$ in two places. The equation looks even cooler. I am intentionally saving numbers for last and going for the elegant formula. I am in theoretical physics mode. Multiply top and bottom by gR.

$$\boxed{\mu_s = \frac{v_0^2 - v^2}{gR + \dfrac{v^2 v_0^2}{gR}}}$$

I believe this form is the coolest! Now I am ready for the numbers: $v_0 = 64.0 \dfrac{\text{km}}{\text{h}}$, $v = 40.0 \dfrac{\text{km}}{\text{h}}$, $R = 200.0 \text{ m}$, and $g = 9.8 \dfrac{\text{m}}{\text{s}^2}$. It would be nice to have the speeds in meters per second.

$$1\frac{\text{km}}{\text{h}} = 1000\frac{\text{m}}{\text{h}} = 1000\frac{\text{m}}{3600 \text{ s}} = \frac{10}{36}\frac{\text{m}}{\text{s}} = \frac{5}{18}\frac{\text{m}}{\text{s}}$$

$$v_0 = 64\frac{\text{km}}{\text{h}} = 64 \cdot \frac{5}{18}\frac{\text{m}}{\text{s}} = 17.7777\frac{\text{m}}{\text{s}}$$

$$v = 40\frac{\text{km}}{\text{h}} = 40 \cdot \frac{5}{18}\frac{\text{m}}{\text{s}} = 11.1111\frac{\text{m}}{\text{s}}$$

$$\mu_s = \frac{v_0^2 - v^2}{gR + \dfrac{v^2 v_0^2}{gR}} = \frac{17.7777^2 - 11.1111^2}{9.8 \cdot 200 + \dfrac{17.7777^2 \cdot 11.1111^2}{9.8 \cdot 200}} = \frac{192.590}{1960 + 19.9072} = 0.097$$

$$\boxed{\mu_s = 0.1}$$

Here are more interesting questions to consider.

3. What is the banking angle?

$$\tan\theta = \frac{v_0^2}{gR}$$

$$\tan\theta = \frac{17.7777^2}{9.8 \cdot 200} = 0.1612$$

$$\boxed{\theta = 9.2°}$$

4. What coefficient of static friction is need to keep a stationary car from sliding down?

$$\boxed{\mu_s = \frac{v_0^2 - v^2}{gR + \dfrac{v^2 v_0^2}{gR}}}$$

Take $v = 0$

$$\mu_s = \frac{v_0^2 - v^2}{gR + \frac{v^2 v_0^2}{gR}} \to \frac{v_0^2 - 0^2}{gR + \frac{0^2 v_0^2}{gR}} = \frac{v_0^2}{gR} = \tan\theta,$$

and note that we already know $\tan\theta = \frac{17.7777^2}{9.8 \cdot 200} = 0.1612$.

$$\mu_s = \frac{v_0^2}{gR} = \tan\theta = 0.16$$

5. With a coefficient of static friction $\mu_s = 0.2$, how fast can you take the curve without skidding.

Now we need to note that for speeds $v > v_0$ the frictional force is down the slanted road. The fastest way to convert our formula is to recognize that wherever μ_s appears, now a minus sign will be in front since the frictional force points the other way. If you are not sure of this shortcut, it is good to work out the problem again. It does not take long since you have already done a similar problem. Therefore

$$\mu_s = \frac{v_0^2 - v^2}{gR + \frac{v^2 v_0^2}{gR}} \quad \text{becomes} \quad -\mu_s = \frac{v_0^2 - v^2}{gR + \frac{v^2 v_0^2}{gR}}.$$

Solve for v.

$$-\mu_s (gR + \frac{v^2 v_0^2}{gR}) = v_0^2 - v^2$$

$$-\mu_s gR - \mu_s \frac{v^2 v_0^2}{gR} = v_0^2 - v^2$$

$$v^2 - \mu_s \frac{v^2 v_0^2}{gR} = v_0^2 + \mu_s gR$$

$$v^2 (1 - \mu_s \frac{v_0^2}{gR}) = v_0^2 + \mu_s gR$$

$$v^2 = \frac{v_0^2 + \mu_s gR}{1 - \mu_s \dfrac{v_0^2}{gR}}$$

$$\boxed{v = \sqrt{\frac{v_0^2 + \mu_s gR}{1 - \mu_s \dfrac{v_0^2}{gR}}}}$$

Data: $v_0 = 64 \dfrac{\text{km}}{\text{h}} = 17.7777 \dfrac{\text{m}}{\text{s}}$, $\mu_s = 0.2$, $g = 9.8 \dfrac{\text{m}}{\text{s}^2}$, $R = 200.0 \text{ m}$.

$$v = \sqrt{\frac{v_0^2 + \mu_s gR}{1 - \mu_s \dfrac{v_0^2}{gR}}} = \sqrt{\frac{17.7777^2 + 0.2 \cdot 9.8 \cdot 200}{1 - 0.2 \dfrac{17.7777^2}{9.8 \cdot 200}}} = \sqrt{\frac{316.047 + 392.000}{1 - 0.03225}} = \sqrt{\frac{708.047}{0.96775}} = \sqrt{731.642}$$

$$v = \sqrt{731.642} = 27.049 \frac{\text{m}}{\text{s}}$$

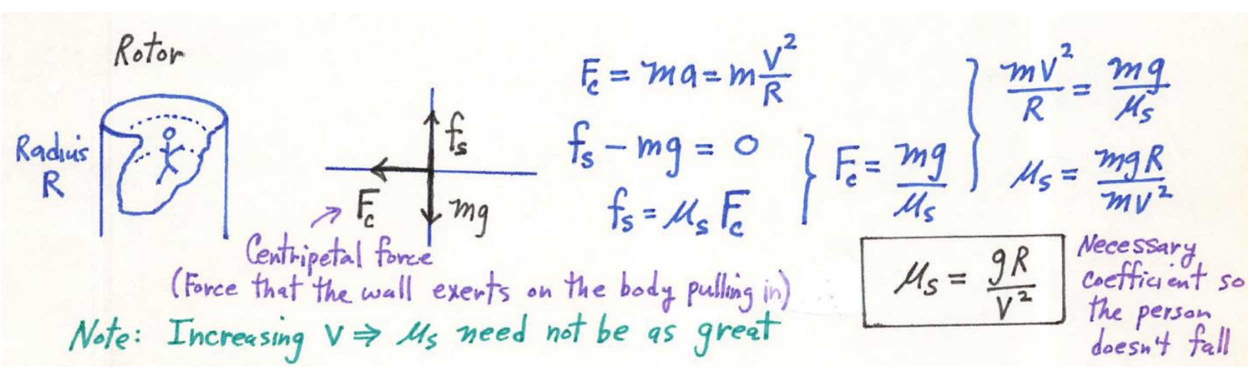
$$v = 27.049 \frac{\text{m}}{\text{s}} = 27.049 \frac{\text{m}}{\text{s}} \cdot \frac{1 \text{ km}}{1000 \text{ m}} \cdot \frac{3600 \text{ s}}{1 \text{ h}} = 27.049 \cdot \frac{18}{5} \frac{\text{km}}{\text{h}} = 97.376 \frac{\text{km}}{\text{h}}$$

$$\boxed{v = 97 \frac{\text{km}}{\text{h}}}$$

$$\boxed{v = 60 \frac{\text{mi}}{\text{h}}}$$

F9. The Rotor Ride. There is an amusement park ride called the rotor. The room rotates and you are help up against the wall by friction when the floor is lowered. You do not fall down!

"Interior of the *Rotor* at Luna Park Sydney. The ride is in mid-cycle, and the riders are stuck to the wall of the barrel by the force of friction combined with their inertia." Courtesy Saberwyn, Wikipedia. License GNU Free.



The analysis for the case where the people are about to budge and fall is above. Here are the steps. The little "c" subscript for the force to the left stands for centripetal. Acceleration due to circular motion is often called centripetal acceleration and the associated force is the centripetal force.

$$F_c = ma = m\frac{v^2}{R}$$

Then you have the force equation in the vertical direction.

$$f_s - mg = 0$$

Finally, we have our friction equation that relates the frictional force to the normal force via the coefficient of static friction. We keep the inequality below.

$$f_s \le \mu_s N$$

But what is the normal force? It is the centripetal force pushing on the backs of the people. The three equations are then

$$F_c = m\frac{v^2}{R}$$

$$f_s = mg$$

$$f_s \le \mu_s F_c$$

The last two equations give

$$mg \le \mu_s F_c$$

Substituting in $F_c = m\frac{v^2}{R}$, we get

$$mg \le \mu_s F_c = \mu_s m\frac{v^2}{R}$$

$$mg \le \mu_s m\frac{v^2}{R}$$

The mass divides out, which is nice, so that the equation applies to all people.

$$g \le \mu_s \frac{v^2}{R} \quad \Rightarrow \quad \frac{gR}{v^2} \le \mu_s \quad \Rightarrow \quad \boxed{\mu_s \ge \frac{gR}{v^2}}$$

Most physics texts might stop here. But let's think like an engineer and do some design work.

These rides can be dangerous. Here is what amusementrideinjurylawyer.com says about the Rotor Ride, also known as the Gravitron.

> "The Rotor or Gravitron ride is a circular chamber that spins around as the floor lowers, causing riders to stick to the wall because of friction and centrifugal force. The scary part about these rides is that they pin riders against a wall with a force that is three times the force of gravity. Rapid spinning and extreme forces of gravity can cause blood in the heart to pump faster to get blood to the brain. This ride may not be suitable for people that suffer from certain medical conditions. The most common accidents that can occur are riders hitting against something or someone during the ride, getting pinched, falling down as spinning subsides, body pain from extreme forces, electric shock, seizures, and equipment failure. Another common occurrence is dizziness, nausea, and vision or hearing problems. These injuries usually occur because of a preexisting condition that a rider had such as a weak heart, asthma, or a neurological condition. Be cautious about riding these types of rides at amusement parks or carnivals if you have a medical condition, especially if these rides lack warning signs." amusementrideinjurylawyer.com

The use of the word "centrifugal" is best replaced with "centripetal." The centrifugal force is the apparent outward force you feel in the rotating frame. Physicists often refer to it as a fictitious force since the real force is the centripetal force making you accelerate, a force directed toward the center of the circular path. We will discuss more about these forces later.

Our basic equation is

$$\boxed{\mu_s \geq \frac{gR}{v^2}}.$$

We can relate this equation to the g-force the person experiences as the wall pushes on them.

Substitute $a = \frac{v^2}{R}$ in the above inequality.

$$\mu_s \geq \frac{g}{a}$$

Now we see that a g-force of 3, i.e., $a = 3g$, gets you a nice low value for μ_s.

$$\mu_s \geq \frac{g}{a} = \frac{g}{3g} = \frac{1}{3}$$

$$\mu_s \geq 0.33$$

This condition is easily met for clothes.

If you design for $a = 2g$, you get

$$\mu_s \geq \frac{g}{a} = \frac{g}{2g} = \frac{1}{2} = 0.5$$

For $a = 1.5g$,

$$\mu_s \geq \frac{g}{a} = \frac{g}{1.5g} = \frac{1}{3/2} = \frac{2}{3} = 0.67 \, .$$

The wall surface can be constructed to be especially rough to achieve this value with normal clothes. If someone comes in with a slick shirt, we will have a jacket ready for that person.

But we do not want folks getting too dizzy. Therefore, I would like for the room to take a full 3 seconds to rotate once. We would do some tests with actual people to make sure. Here is the equation that relates the speed to the time to go around once. We call this time the period and we will use T for it.

$$v = \frac{2\pi R}{T}$$

We want $T = 3.0 \text{ s}$. In this way, the room rotates once in three seconds.

$$a = \frac{v^2}{R} = (\frac{2\pi R}{T})^2 \frac{1}{R} = \frac{4\pi^2 R^2}{T^2} \frac{1}{R}$$

$$a = \frac{4\pi^2 R}{T^2}$$

For $a = \frac{3}{2} g$, we can see how large the rotating cylinder radius has to be?

$$a = \frac{4\pi^2 R}{T^2} = \frac{3}{2} g$$

$$R = \frac{3}{2} g \frac{T^2}{4\pi^2}$$

$$R = \frac{3}{2} (9.8 \frac{\text{m}}{\text{s}^2}) \frac{(3.0 \text{ s})^2}{4\pi^2} = \frac{3}{2} \cdot \frac{9.8}{\pi^2} \cdot \frac{9}{4} = 3.35$$

$$R = 3.4 \text{ m}$$

The Rotor at Luna Park Sydney
New South Wales, Australia
Courtesy Saberwyn, Wikipedia.
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The radius at the left looks more like 2 m. I used the adults as yard sticks and figured if they were lying out on the floor towards the center, the radius would be about 2 meters. I found a YouTube video that shows the rotation. I counted 10 rotations for 18 seconds. Therefore $T = \frac{18 \text{ s}}{10} = 1.8 \text{ s}$.

We can now calculate the g-force.

$$a = \frac{v^2}{R} = \left(\frac{2\pi R}{T}\right)^2 \frac{1}{R} = \frac{4\pi^2 R^2}{T^2} \frac{1}{R}$$

$$a = \frac{4\pi^2 R}{T^2}$$

$$a = \frac{4\pi^2 R}{T^2 g} g$$

$$a = \frac{4\pi^2 (2 \text{ m})}{(1.8 \text{ s})^2 (9.8 \frac{\text{m}}{\text{s}^2})} g$$

$$a = \frac{4(3.14159)^2 (2)}{(1.8)^2 (9.8)} g$$

$$a = \frac{78.957}{31.752} g$$

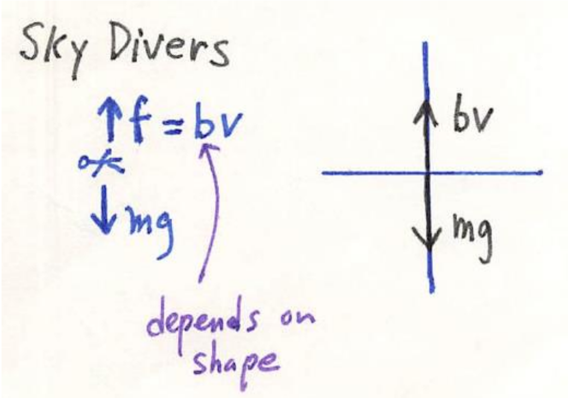
$$a = 2.49g$$

$$a = 2.5g$$

F10. Skydiving. Here we consider a skydiver, where air resistance serves as a retarding force. Such resistance is also called drag. Unlike surface-to-surface contact, the drag depends on the speed. For speeds that are not too high, like sky diving, the drag is approximately proportional to the speed.

Skydiver Courtesy Добромир Славчев, Wikimedia. CC0 1.0 Universal Public Domain Dedication

Question. What is the terminal velocity in terms of the mass m of the skydiver, g due to gravity, and the air resistance constant b where the resistance force is f = bv?



Solution.

(i) Sketch. See left drawing of falling skydiver.

(ii) Force Diagram. There are two forces on the skydiver: the gravitational weigh W = mg and the air resistance force f = bv, proportional to the velocity at these speeds.

(iii) Newton's Law. I will choose down as positive:

$$mg - bv = ma = 0\,,$$

since when terminal velocity is reached, there is no longer any acceleration. The terminal velocity is then

$$\boxed{v_t = \frac{mg}{b}}.$$