Physics I with Calculus, Prof. Ruiz (Doc), UNC-Asheville (1978-2021), DoctorPhys on YouTube
Chapter E. Dynamics. Prerequisite: Calculus I. Corequisite: Calculus II.

E0. Statics and Dynamics. In this chapter we study dynamics. In the last chapter we introduced statics. Here are definitions of each of these subject areas.

Statics – the study of forces and their effects on a body at rest.

Dynamics – the study of forces and their effects on a body in motion.

We can define these in terms of Newton's Second Law $\vec{F} = m\vec{a}$.

Statics. The net force on a body is zero. $\vec{F} = 0$
Later we will add that torques (forces acting to rotate an object) are also zero.

Dynamics. The net force on a body is nonzero. $\vec{F} = m\vec{a} \neq 0$

These topics are so important in engineering that there is an entire course on each one in Engineering Departments. Below are the course descriptions of our course (minus the lab), statics, and dynamics. For 7 years from the academic years 1984-1985 to 1990-1991 I got to teach North Carolina State's Statics and Dynamics courses here in Asheville as part of our Two-Plus-Two Program with *North Carolina State University* in Raleigh. We did the statics in the Fall and the dynamics in the Spring. I gained a great respect for engineers when I learned and then taught methods to calculate a force in one of the beams in a bridge truss. There was plenty of material to fill a semester with applications of $\vec{F} = 0$.

PHYS 221 Physics I (4 credit hours). Introductory calculus-based physics for science and engineering students with laboratory covering Newtonian statics and dynamics, fluids, heat and sound.

ENGR 206 Engineering Statics (3 credit hours). Basic concepts of forces in equilibrium. Distributed forces, frictional forces. Inertial properties. Application to machines, structures, and systems.

ENGR 208 Engineering Dynamics (3 credit hours). Kinematics and kinetics of particles in rectangular, cylindrical, and curvilinear coordinate systems; energy and momentum methods for particles; kinetics of systems of particles; kinematics and kinetics of rigid bodies in two and three dimensions; motion relative to rotating coordinate systems.

A bridge truss is shown on the next page. In ENGR 206 Statics we learn how to calculate the forces in truss beams and tell whether they are compressions (squeezing the beam) or tensions (stretching the beam).

A Southern Pacific Railroad bridge, now part of the Iron Horse Regional Trail in Contra Costa County, California. Wikimedia. Photo Courtesy Leonard G., released into the Public Domain.

Star 266-based cherry picker during reconstruction of Długa street in Kraków. Courtesy SuperTank17, Wikimedia Creative Commons CC BY-SA License

For ENGR 208 Dynamics one learns how to calculate motions of beams such as those shown in the Cherry Picker at the left. In the truss photo above we have stationary beams. For the cherry picker we have beams in motion.

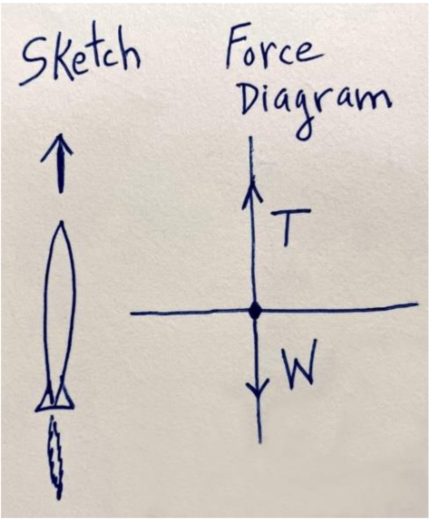
In our course we will introduce you to basic problems in dynamics. The engineering statics and dynamics courses build on the foundation you acquire in our introductory physics course.

The steps for solving statics and dynamics problems are the same 1)sketch a picture, 2)give a force diagram, 3)apply Newton's Law, 4)solve.

Let's do 10 dynamics problems!

E1. Apollo 15 Rocket. The Apollo 15 spacecraft (weight about $0.48\times10^6\ \text{N} = 0.11\times10^6\ \text{lb}$) sits on the massive Saturn V rocket (about $27.1\times10^6\ \text{N} = 6.09\times10^6\ \text{lb}$). The rocket thrust at the very early phase of the trip is about $33.4\times10^6\ \text{N}$ ($7.5\times10^6\ \text{lb}$) and **you can neglect any loss of rocket mass at this point.** How long does it take the rocket to clear the 136 meter (445 ft) tower?

Launch of Apollo 15. July 26, 1971. Photo Courtesy NASA, Public Domain.



(i) The Sketch (see left).

(ii) The Force Diagram. The rocket is the point at the origin. There is the force pulling the rocket down due to gravity. It is labeled W for weight. The upward force is the thrust.

(iii) Newton's Second Law. We now apply $F = ma$ in the vertical direction. The net force F = T – W.

$$F = T - W = ma$$

(iv) Solve. We want the acceleration: $a = \dfrac{T - W}{m}$. But what is m? We get the mass from the weight since $W = mg$, i.e., the force on the mass due to gravity. The weight to be lifted is the sum of the weight of the Apollo 15 capsule at the top plus the weight of the Saturn V.

$$W = 27.1 \times 10^6 \text{ N} + 0.48 \times 10^6 \text{ N} = 27.58 \times 10^6 \text{ N}$$

The total mass is given by

$$m = \frac{W}{g} = \frac{27.58 \times 10^6 \text{ N}}{9.8 \text{ m/s}^2} = 2.814 \times 10^6 \text{ kg} .$$

Now we are ready to calculate the acceleration. We use

$$a = \frac{T - W}{m} ,$$

where the thrust $T = 33.4 \times 10^6 \text{ N}$, weight $W = 27.58 \times 10^6 \text{ N}$, and mass $m = 2.814 \times 10^6 \text{ kg}$.

$$a = \frac{T - W}{m} = \frac{33.4 \times 10^6 \ - 27.58 \times 10^6}{2.77 \times 10^6} = \frac{33.4 - 27.1}{2.77} = \frac{6.3}{2.77} = 2.068 \ \frac{\text{m}}{\text{s}^2}$$

$$a = 2.07 \ \frac{\text{m}}{\text{s}^2}$$

To find the time to clear the tower, use

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 ,$$

where

$y_0 = 0$ (the rocket starts at the ground),

$y = 136 \text{ m}$ (the bottom of the rocket clears the tower),

$v_0 = 0$ (the rocket starts out from rest),

$a = 2.07 \ \frac{\text{m}}{\text{s}^2}$ (the upward acceleration).

$$y = \frac{1}{2} a t^2$$

$$\frac{2y}{a} = t^2$$

$$t = \sqrt{\frac{2y}{a}}$$

$$t = \sqrt{\frac{2 \cdot 136}{2.07}} = \sqrt{\frac{272}{2.07}} = \sqrt{131.40} = 11.46 = 11 \text{ s}$$

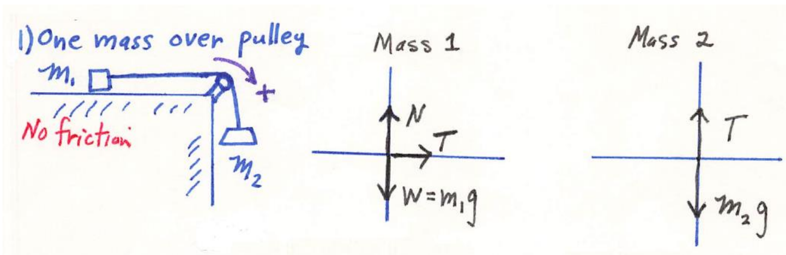
$$\boxed{t = 11 \text{ s}}$$

Always remember to include “Reflection” of your answer. Is the answer reasonable?. Also pay attention to the good “Communication” with the right amount of significant figures and proper units in the final answer. Also note that our kinematic formulas are used in dynamics problems.

I found and watched the video, and indeed, the tower is cleared in about this time!

E2. Pulley and Table. In this section we do a pulley problem, where there are two masses to deal with. So we will need two force diagrams, one for each mass.

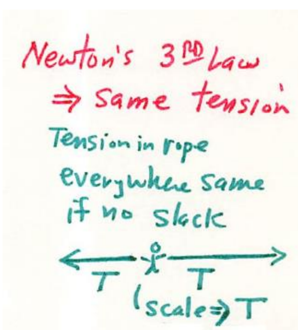
Mass m_2 is pulling mass m_1 across the table. There is no friction anywhere. This no-friction statement means that m_1 has no friction when it slides on the table and the rope that wraps around the pulley has no friction with the pulley. Think of the rope as slipping over a stationary pulley (no rotation of the pulley) and think of m_1 as sliding on ice. Find formulas for the acceleration (a) and tension (T).



We have (i) our sketch at the left and (ii) our force diagrams. Note that we need two, one for each mass.

Mass m_1 has three forces on it: the force $W = m_1g$ that pulls it down due to gravity, the force of the table upward which is called the normal force N, and the rope force T to the right.

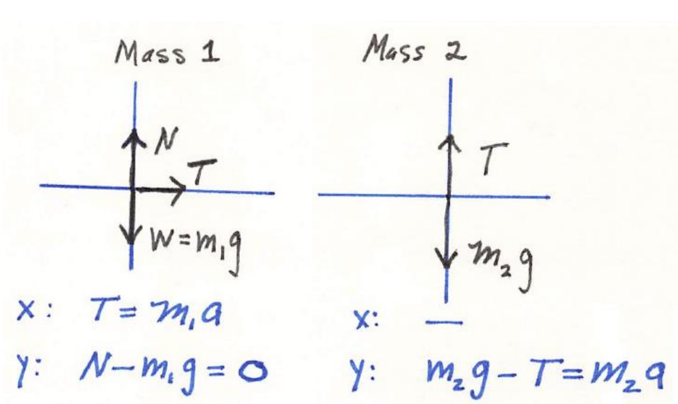
Mass m_2 has two forces acting on it: the weight m_2g pulling it down and the rope force T pulling it upward.



Note that we have invoked Newton's Third Law of Action and Reaction. As m_2 pulls m_1 with a force designated by T (the action), m_1 pulls back on m_2 with an equal and opposite force having the same magnitude T.

We also assume in our problems taut ropes so there is no slack anywhere. Therefore, if you were to cut the rope at any point and grab each cut end or stick a scale in there, you would feel or measure the same tension T.

We are ready for step (iii) Newton's Second Law. Now it is time to note which direction is positive.



I made a choice in the above figure. If the problem doesn't say, then you pick the positive direction. Since mass 2 is pulling mass 1, that means mass 2 will move down. So that is my choice for the positive direction for mass 2. Then, to be consistent, the positive direction for mass 1 is to the right. I need to remember the plus and minus direction for each mass when I proceed to write down the force equations. Applying F = ma for each direction (x and y) for m_1 gives two equations. We only have the y equation for m_2.

Note that the acceleration of each mass is the same as they move together. The three equations are given below.

$$T = m_1 a$$

$$N - m_1 g = 0$$

$$m_2 g - T = m_2 a$$

The equation with the normal force N just tells us that $N = m_1 g$, i.e., the table exerts an upward force equal to the weight of the mass on the table since that mass is in equilibrium with respect to the vertical direction. It neither moves up or down. It stays on the table.

So that leaves two equations.

$$T = m_1 a$$

$$m_2 g - T = m_2 a$$

But we are in good shape. We have two equations with two unknowns, the T and a, which is what the problem asks us to solve for.

I like this kind of problem that does not have numbers. We are deriving general equations for the pulley configuration. If you enjoy this kind of problem, you are leaning towards theoretical physics. If you really prefer getting numbers in there as soon as possible, you are leaning towards engineering. If you like both, you are the best of both worlds. Though I am a theoretical physicist, I enjoy both very much. Teaching statics and dynamics was lots of fun.

Solving two equations for two unknows is a standard high school algebra problem. Since you have T in the first equation, you can substitute it into the second equation. But I like to approach the solution in a different way that comes in handy lots of times. Thanks to Newton's Third Law of Action and Reaction, the tension T appears in the second equation with opposite sign. The physics is suggesting to add the equations and thus eliminate one of the unknowns immediately that way. Doing this, we get

$$T + m_2 g - T = m_1 a + m_2 a$$

$$m_2 g = (m_1 + m_2) a$$

$$\boxed{a = \left(\frac{m_2}{m_1 + m_2}\right) g}$$

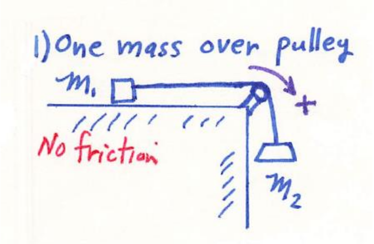
To find the tension T, use the above equation with $T = m_1 a$. Then, $T = m_1 a = m_1 (\frac{m_2}{m_1 + m_2}) g$.

$$\boxed{T = (\frac{m_1 m_2}{m_1 + m_2}) g}$$

Time for checking on the answers. Remember our I-ARC guide: Inquire, Apply, Reflection, and Communicate. We were given the question (Inquire) and we applied physics to solve the problem. What about Reflection and Communicate?

In other words, are our answers reasonable (Reflection) and do they have the correct dimensions (Communication). The acceleration equation $a = (\frac{m_2}{m_1 + m_2}) g$ has the correct dimensions since the ratio of the masses $\frac{m_2}{m_1 + m_2}$ is dimensionless and we are left with the dimensions of g, which is acceleration. For the T equation $T = (\frac{m_1 m_2}{m_1 + m_2}) g$, we have the dimensionless $\frac{m_2}{m_1 + m_2}$ factor and then there is an extra m_1 factor, which when its units hit the g units, you get force units. Remember that the force due to gravity pulling on a mass is given by mg near the Earth's surface.

What about the reasonableness of the answers $\boxed{a = (\frac{m_2}{m_1 + m_2}) g}$ and C?



Check 1. What happens if m_1 is extremely large, going off to infinity? Well, it won't move then and m_2 will just hang there. Let's see what the equations tell us.

$$\lim_{m_1 \to \infty} a = \lim_{m_1 \to \infty} (\frac{m_2}{m_1 + m_2}) g = 0$$

$$\lim_{m_1 \to \infty} T = \lim_{m_1 \to \infty} (\frac{m_1 m_2}{m_1 + m_2}) g = \lim_{m_1 \to \infty} (\frac{m_2}{1 + \frac{m_2}{m_1}}) g = (\frac{m_2}{1+0}) g = m_2 g$$

Note the trick of dividing the numerator and denominator by m_1 in evaluating the limit.

These results are the expected ones. The mass m_1 on the table does not move and the tension in the rope is due to the weight of the hanging mass m_2.

Check 2. Mass m_2 is super large. We expect mass m_2 to fall unimpeded at $a = g$. Since the acceleration is the same for both masses, m_2 will also accelerate at $a = g$ and the force on it by Newton's Second Law has to be $F_1 = m_1 a = m_1 g$. Let's see if the formulas agree.

$$\lim_{m_2 \to \infty} a = \lim_{m_2 \to \infty} (\frac{m_2}{m_1 + m_2}) g = \lim_{m_2 \to \infty} (\frac{1}{\frac{m_1}{m_2} + 1}) g = (\frac{1}{0+1}) g = g$$

$$\lim_{m_2 \to \infty} T = \lim_{m_2 \to \infty} (\frac{m_1 m_2}{m_1 + m_2}) g = \lim_{m_2 \to \infty} (\frac{m_1}{\frac{m_1}{m_2} + 1}) g = (\frac{m_1}{0+1}) g = m_1 g$$

Check 3. Mass m_1 is extremely small. We expect $a = g$ with no tension in the rope, i.e., $T = 0$.

$$\lim_{m_1 \to 0} a = \lim_{m_1 \to \infty} (\frac{m_2}{m_1 + m_2}) g = (\frac{m_2}{0 + m_2}) g = \frac{m_2}{m_2} g = g$$

$$\lim_{m_1 \to 0} T = \lim_{m_1 \to \infty} (\frac{m_1 m_2}{m_1 + m_2}) g = (\frac{0 \cdot m_2}{0 + m_2}) g = 0$$

Check 4. Mass m_2 is super small. It will then just hang there incapable of making m_1 move. We expect $a = 0$ with a tension in the rope equal to the weight of m_2, i.e., $T = m_2 g \approx 0$. Let's see.

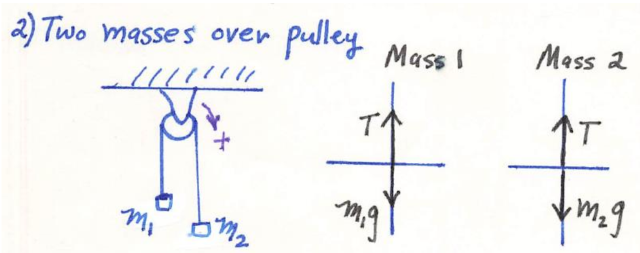
$$\lim_{m_2 \to 0} a = \lim_{m_2 \to 0} (\frac{m_2}{m_1 + m_2}) g = 0$$

$$\lim_{m_2 \to small} T = \lim_{m_2 \to small} (\frac{m_1 m_2}{m_1 + m_2}) g = \lim_{m_2 \to small} (\frac{m_2}{1 + \frac{m_2}{m_1}}) g = m_2 g$$

$$\lim_{m_2 \to 0} T = \lim_{m_2 \to \infty} (\frac{m_1 m_2}{m_1 + m_2}) g = 0$$

An engineer that is designing a system has to think in terms of variables like this, similar to a theoretical physicist. Here is an area where engineers and physicists think alike.

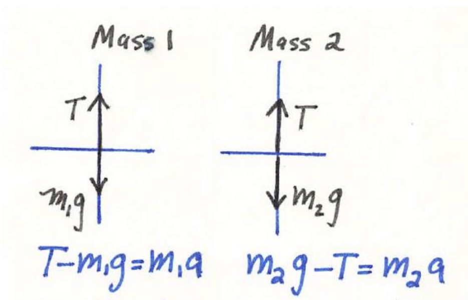
E3. Two Masses Over a Pulley. Here we have two masses hanging where the rope goes over a pulley. Once again, there is no friction. The rope can slip easily over the pulley. Again, we are to find the acceleration a and tension T.



The (i) sketch is at the left along with the (ii) force diagram. Such force diagrams are also called **free body diagrams** since the masses are placed at the origin free from the structure. But we do include all the forces.

Note the importance of choosing a positive direction. See the "+" sign in the sketch. That choice means for positive acceleration m_1 will move up and m_2 will move down, For negative accelerations, the masses will move the opposite way.

(iii) Newton's Second Law. The two equations are below.



$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$

(iv) Solve. Here is where Newton's Third Law leads the way. The tension T appears with opposite signs in the equations due to the Action-Reaction feature. So the physics is suggesting we add the equations as we did in the previous pulley problem.

$$T - m_1 g + m_2 g - T = m_1 a + m_2 a$$

The tensions cancel.

$$-m_1 g + m_2 g = m_1 a + m_2 a$$

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$\boxed{a = (\frac{m_2 - m_1}{m_1 + m_2})g}$$

We can now substitute a into any of the two force equations. I like the first one.

$$T - m_1 g = m_1 a$$

$$T - m_1 g = m_1 (\frac{m_2 - m_1}{m_1 + m_2}) g$$

$$T = m_1 g + m_1 (\frac{m_2 - m_1}{m_1 + m_2}) g$$

$$T = m_1 \left[1 + (\frac{m_2 - m_1}{m_1 + m_2}) \right] g$$

$$T = m_1 \left[\frac{m_1 + m_2}{m_1 + m_2} + \frac{m_2 - m_1}{m_1 + m_2} \right] g$$

$$T = m_1 (\frac{m_1 + m_2 + m_2 - m_1}{m_1 + m_2}) g$$

$$T = m_1 (\frac{2m_2}{m_1 + m_2}) g$$

$$\boxed{T = (\frac{2m_1 m_2}{m_1 + m_2}) g}$$

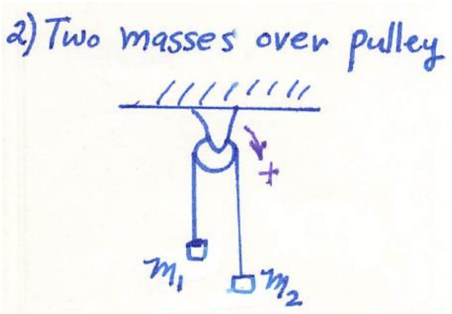
Communication Check. Do the units make sense? Are they correct?

Yes for $\boxed{a = (\frac{m_2 - m_1}{m_1 + m_2}) g}$ since the mass units cancel and a then has units of g. Correct!

Yes for $\boxed{T = (\frac{2m_1 m_2}{m_1 + m_2}) g}$ since the units in parentheses reduce to mass and Mg gives force.

Reflection Check. Do the answers make sense? You can consider the “Communication Check” as a “Reflection” but I like to separate them. For “reflection” I ask myself if the answers make sense. For “communication” I ask myself are the units correct and if I am using numbers, do I have the right amount of significant figures?

Check 1. What happens if m_1 is extremely large, going off to infinity?



$$\lim_{m_1\to\infty} a = \lim_{m_1\to\infty} (\frac{m_2 - m_1}{m_1 + m_2})g$$

$$\lim_{m_1\to\infty} a = \lim_{m_1\to\infty} \left[\frac{\frac{m_2}{m_1} - 1}{1 + \frac{m_2}{m_1}}\right] g = \left[\frac{0-1}{1+0}\right] g = -g$$

The acceleration is negative, which means that m_1 moves down. And it also falls at the usual acceleration due to gravity without being impeded. The mass m_2 is so light relative to m_1 that it has no effect on the falling m_1. What about the tension in the rope?

$$\lim_{m_1\to\infty} T = \lim_{m_1\to\infty} (\frac{2m_1 m_2}{m_1 + m_2})g = \lim_{m_1\to\infty} (\frac{2m_2}{1 + \frac{m_2}{m_1}})g = 2m_2 g$$

What's with that? It makes sense since if m_2 were at rest hanging, you would have $m_2 g$. Now if you accelerate it upward at g, you need an extra $m_2 g$.

Check 2. What happens if m_2 is extremely large, going off to infinity? We expect that m_2 will fall freely pulling m_1 up. The acceleration will be a positive g for each mass. Let's see if the math comes out right?

$$\lim_{m_2\to\infty} a = \lim_{m_2\to\infty} (\frac{m_2 - m_1}{m_1 + m_2})g$$

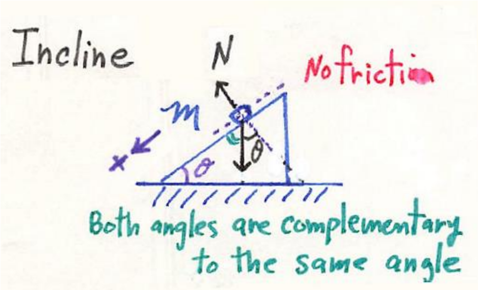
$$\lim_{m_2\to\infty} a = \lim_{m_2\to\infty} \left[\frac{1 - \frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right] g = \left[\frac{1-0}{0+1}\right] g = g$$

It checks out.

For the tension, we get a similar result as before, but now with m_1 instead of m_2.

$$\lim_{m_2\to\infty} T = \lim_{m_2\to\infty} (\frac{2m_1 m_2}{m_1 + m_2})g = \lim_{m_1\to\infty} (\frac{2m_1}{\frac{m_1}{m_2} + 1})g = 2m_1 g$$

E4. Block on Incline with No Friction. A block of mass m in on an incline that makes an angle θ with respect to the horizontal. There is no friction. Find the acceleration of the mass.



At the left is (i) a sketch and (ii) the force diagram superimposed on it. There are two forces acting on the mass, the weight due to gravity pulling down and the force the incline exerts normal to the block. There has to be this normal force; otherwise, the mass would penetrate into the incline.

(iii) Newton's Laws. Along the incline: $mg\sin\theta = ma$

Along the normal force: $N - mg\cos\theta = 0$

These equations lead immediately to the following. The answer to the question for the acceleration is given by the first equation.

$$\boxed{a = g\sin\theta}$$

$$\boxed{N = mg\cos\theta}$$

Communication Check. Are the units correct? Yes since g has acceleration units and mg has force units.

Reflection Check. Are the answers reasonable?

Check 1. The case where the angle is zero.

$$\lim_{\theta\to 0^\circ} a = \lim_{\theta\to 0^\circ}(g\sin\theta) = g\sin(0^\circ) = 0$$

$$\lim_{\theta\to 0^\circ} N = \lim_{\theta\to 0^\circ}(mg\cos\theta) = mg\cos(0^\circ) = mg$$

These results are expected. At no incline, there is no acceleration and the normal force pushing up on the block is equal to the weight of the block.

Check 2. The case where the angle is 90°.

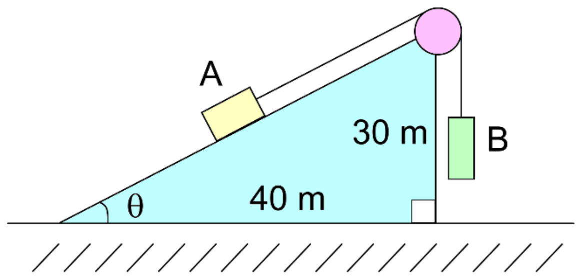
$$\lim_{\theta\to 90^\circ} a = \lim_{\theta\to 90^\circ}(g\sin\theta) = g\sin(90^\circ) = g$$

$$\lim_{\theta\to 90^\circ} N = \lim_{\theta\to 90^\circ}(mg\sin\theta) = mg\sin(0^\circ) = 0$$

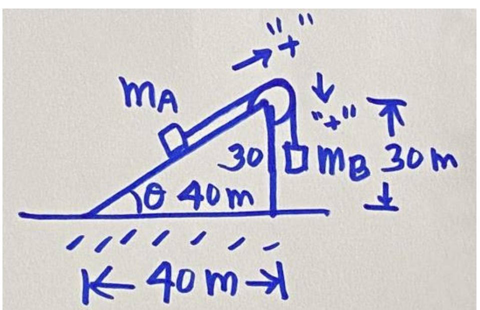
These results are expected again. The block is in free fall and there is no normal force.

E5. Incline with Pulley. Object A weighs 10 N and rests on a frictionless plan with dimensions shown in the figure. Object B weighs 20 N. The masses are held in place, then released.

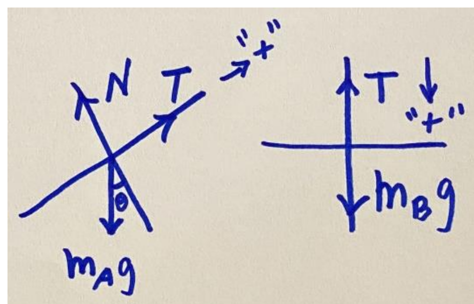
1. After release, what is the acceleration of the masses in terms of g?
2. After release, does A move up the plane or down?
3. What is the speed of each mass 1 second after release?



(i) Sketch



(ii) Force Diagrams



We choose up the incline as positive. If $a < 0$, that means mass A is moving down the incline.

(iii) Newton's Laws.

Mass A: $T - m_A g \sin\theta = m_A a$

Mass B: $m_B g - T = m_B a$

Add: $m_B g - m_A g \sin\theta = (m_A + m_B) a$ => $a = \dfrac{m_B g - m_A g \sin\theta}{m_A + m_B}$ => $a = \left[\dfrac{m_B g - m_A g \sin\theta}{(m_A + m_B) g}\right] g$

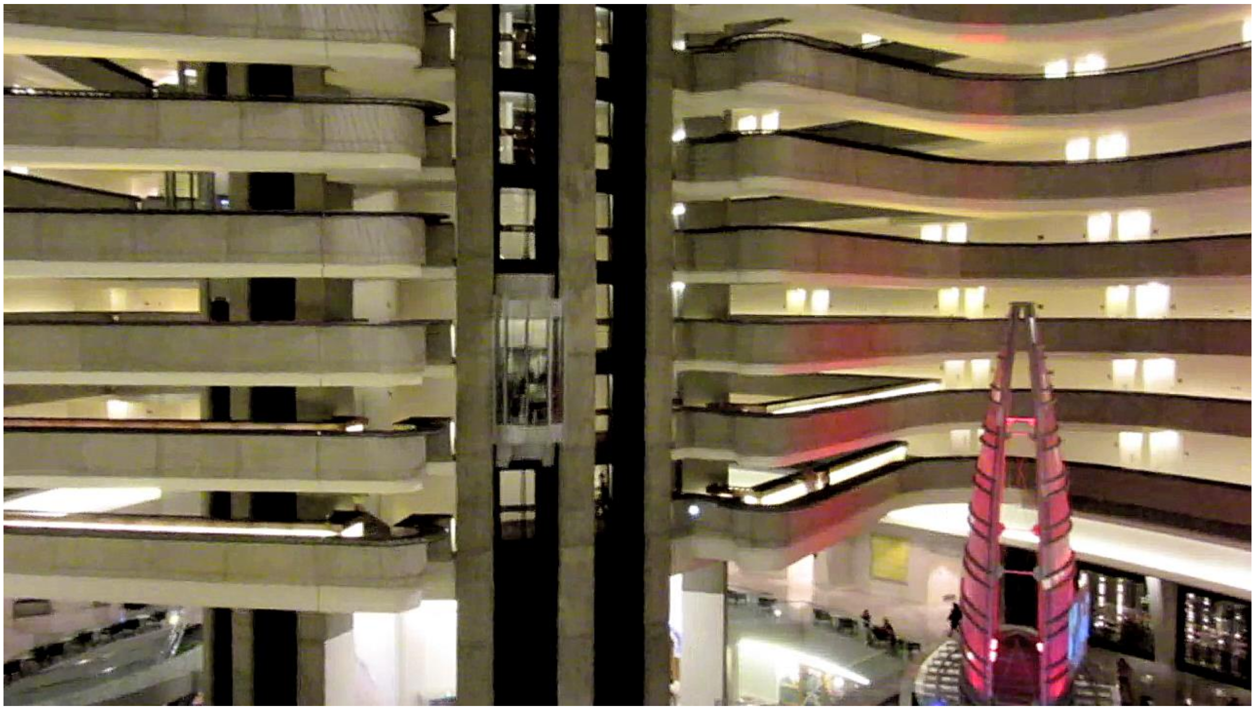
$$1.\ a = \left[\frac{m_B g - m_A g \sin\theta}{m_A g + m_B g}\right] g = \frac{20 - 10(3/5)}{10 + 20} g = \frac{20-6}{30} g = \frac{14}{30} g = \frac{7}{15} g$$

2. Mass A moves up the incline since $a > 0$.

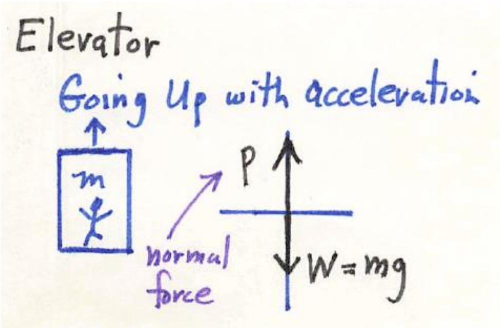
3. Use $v = v_0 + at$, where $v_0 = 0$, $a = \dfrac{7}{15} g = \dfrac{7}{15} \cdot 9.8 \dfrac{\text{m}}{\text{s}^2} = 4.573 \dfrac{\text{m}}{\text{s}^2}$, and $t = 1\ \text{s}$.

$$v = 0 + at = 4.573 \frac{\text{m}}{\text{s}^2} \cdot 1\ \text{s} = 4.6 \frac{\text{m}}{\text{s}}$$

E6. The Elevator. An elevator is accelerating upward as a cable exerts an upward force P. Give your apparent weight if you are standing on a scale in the elevator. What about accelerating downward? What about decelerating but moving updward?



Glass Elevator at the Atlanta Marriott Marquis Hotel, Atlanta, Georgia, USA
Photo by doctorphys, February 17, 2017



1. Going Up. The upward force P is transmitted so that it is also the normal force upward on your feet due to the floor of the elevator.

$$P - mg = ma$$

$$P = ma + mg = m(a + g)$$

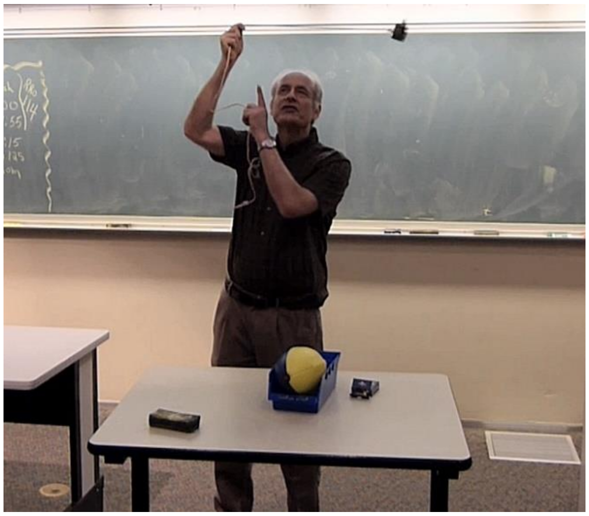
$$\boxed{P = m(g + a)}$$

The scale will read P. For a > 0 the scale shows a value greater than your weight: $P > mg$.

2. Going Down. For downward acceleration, a < 0, the scale reads less: $P < mg$. You can also be going upward, but with deceleration occuring. Then a < 0 for that case also.

Note that if $a = -g$, then you are weightless, i.e., in free fall and the scale reads zero: $P = 0$.

E7. Twirling a Mass on a String. I am twirling a mass with a buzzer to demonstrate the changes in pitch known as the Doppler Effect. A photo of the demonstration is below. The class was *The Physics of Sound and Music* on September 3, 2015.



Question. What is the tension in the string if I make two complete revolutions per second, the mass is 0.1 kg, and the radius of the circle is 0.7 m?

Solution. Here is our first problem with acceleration due to circular motion. **Note that for circular motion, the acceleration is towards the center of the circle.**

$$T = \frac{mv^2}{r}$$

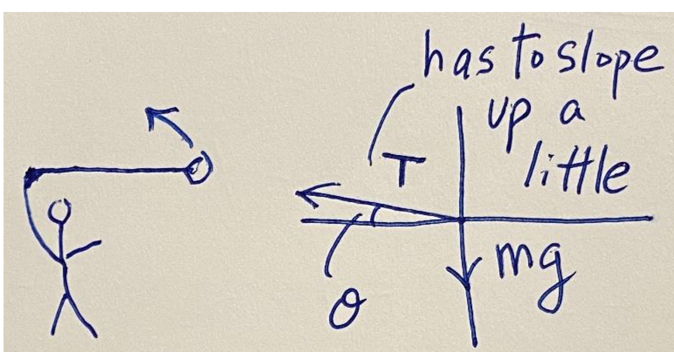
$$v = \frac{2\pi r}{t} = \frac{2\pi \cdot (0.7 \text{ m})}{0.5 \text{ s}} = 8.7965 \frac{\text{m}}{\text{s}}$$

$$T = \frac{mv^2}{r} = \frac{(0.1 \text{ kg})(8.7965 \text{ m/s})^2}{0.7 \text{ m}} = 11.05 \text{ N} = 11 \text{ N}$$

$$\boxed{T = 11 \text{ N}}$$

This tension $T = 11 \text{ N} \frac{1 \text{ lb}}{4.45 \text{ N}} = 2.5 \text{ lb}$

Where is the force diagram and what is holding the mass up in the air?



The string has to slant up a little to get the forces balanced in the vertical direction. Look carefully at the above photo and you will see a slight angle. I didn't give the force diagram first because I didn't want you worrying about this subtlety. But let's revisit things again now more carefully.

$$T\cos\theta = \frac{mv^2}{r} \qquad T\sin\theta = mg$$

Let's see what this angle is. Divide the equation $T\sin\theta = mg$ by the equation $T\cos\theta = \frac{mv^2}{r}$.

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mg}{mv^2/r} = \frac{gr}{v^2}$$

$$\tan\theta = \frac{gr}{v^2}$$

We know $g = 9.8\ \frac{\text{m}}{\text{s}^2}$, $r = 0.7\ \text{m}$ was given, and we calculated $v = 8.7965\frac{\text{m}}{\text{s}}$.

$$\tan\theta = \frac{gr}{v^2} = \frac{(9.8)\cdot(0.7)}{8.7965^2}$$

$$\tan\theta = 0.08866$$

$$\theta = \tan^{-1} 0.08866 = 5.0666°$$

$$\theta = 5°$$

This angle is very, very small.

Now we should get the tension since we are doing the problem more accurately.

$$T\sin\theta = mg$$

$$T = \frac{mg}{\sin\theta}$$

The mass was given as $m = 0.1\ \text{kg}$.

$$T = \frac{mg}{\sin\theta} = \frac{(0.1)(9.8)}{\sin 5.0666°} = \frac{0.98}{0.0883} = 11.097$$

$$T = 11\ \text{N}$$

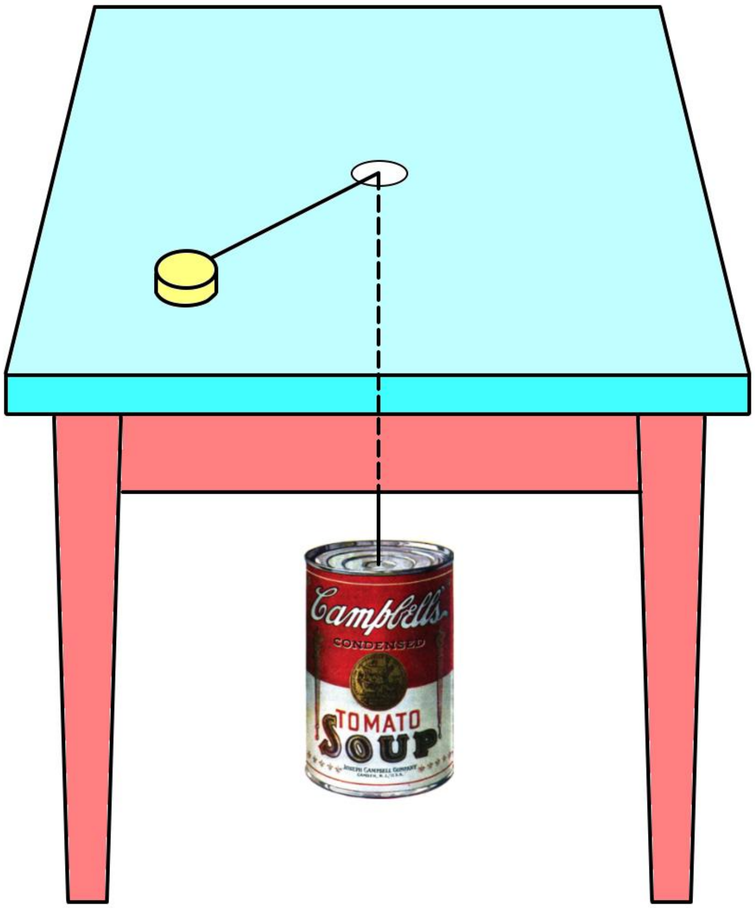
This result is the same as before to two significant figures.

E8. Revolving Mass on a Table. A mass (m) is set into a circular motion on a table with an attached string pulled taut due to a hanging mass (M).

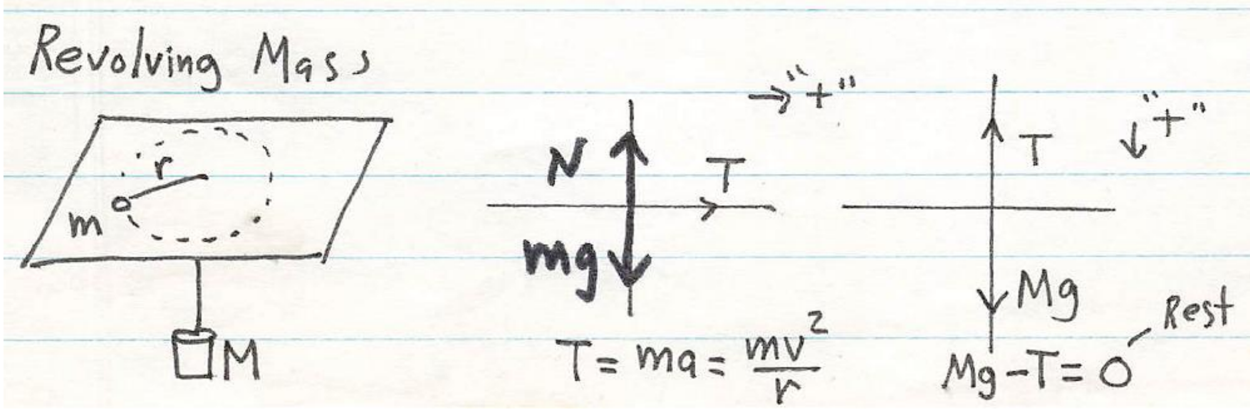
The hanging mass is a can of Campbell's Soup chosen by your author since he is from Camden, New Jersey, home of Campbell Soup.

If you ever see Campbell Soup in a grocery store, pick up a can and look on the back. Do you see Camden, NJ printed somewhere?

Problem. Derive an equation for the velocity needed in terms of the mass of the little yellow puck (m), the mass of the soup can (M), the radius of the circular orbit (r), and the gravitational constant (g).



A sketch and forces diagrams are below. The left force diagram is for mass m (the puck) and the right force diagram is for mass M (the soup can). The equations appear below each diagram.



The equations for the puck are $N = mg$ and $T = m\frac{v^2}{r}$. For the soup we get $T = Mg$. The tension in the string is T and Newton's Third Law of Action-Reaction has been applied. A positive direction has also been chosen in a consistent fashion for the masses m and M.

The equations we need to finish the problem are these two:

$$T = m\frac{v^2}{r} \quad \text{and} \quad T = Mg\,.$$

We are asked to solve for the velocity v. We first eliminate T.

$$m\frac{v^2}{r} = Mg$$

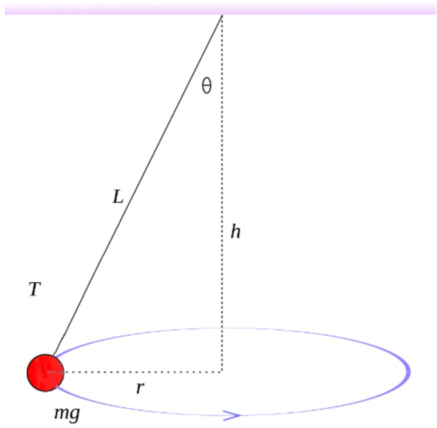
Now solve for v.

$$\frac{v^2}{r} = \frac{Mg}{m}$$

$$v^2 = \frac{Mgr}{m}$$

$$\boxed{v = \sqrt{\frac{Mgr}{m}}}$$

Note that for circular motion, the acceleration is towards the center of the circle.

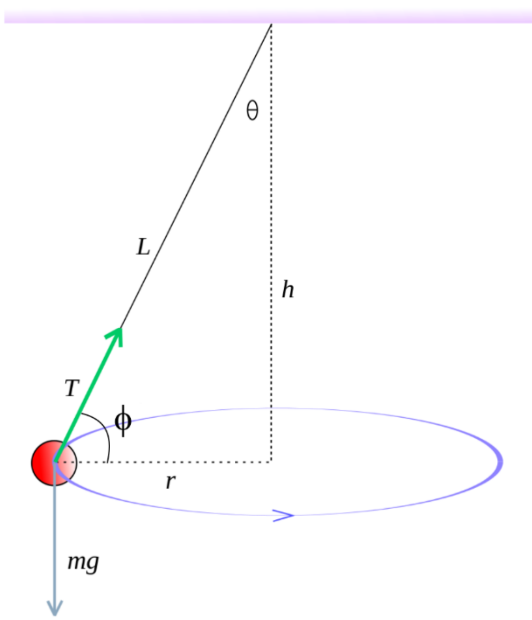


E9. Conical Pendulum. A conical pendulum bob is swinging in a horizontal circle as shown in the figure (Courtesy CosineKitty, Wikimedia, Public Domain). The bob mass is m, the length of the pendulum cable is L, the vertical length is h, the radius of the circle is r, the angle is θ, the circular speed of the bob is v, and the tension in the pendulum cable is T.

1. Find the angle θ in terms of m, v, r, and g. Note that the angle might be independent of one or more of these parameters.

2. Find the specific angle when $m = 3.6\ \text{kg}$, $v = 2.1\ \dfrac{\text{m}}{\text{s}}$, and $r = 1.2\ \text{m}$.

3. Find L in meters for the data given in 2.



Solution. For this problem we are merging the sketch and force diagram in the figure at the left.

$$\sum F_x = ma \quad => \quad T\cos\phi = m\frac{v^2}{r}$$

$$\sum F_y = T\sin\phi - mg = 0$$

Since $\theta + \phi = 90°$.

$$T\sin\theta = m\frac{v^2}{r}$$

$$T\cos\theta = mg$$

Divide the equations to get $\tan\theta = \dfrac{v^2}{gr}$.

1. The angle formula: $\theta = \tan^{-1}(\dfrac{v^2}{gr})$. Compare this formula to the angle formula in E7?

2. The angle with given numbers: $\theta = \tan^{-1}(\dfrac{2.1^2}{9.8 \cdot 1.2}) = \tan^{-1}(0.37500) = 20.556° = 21°$.

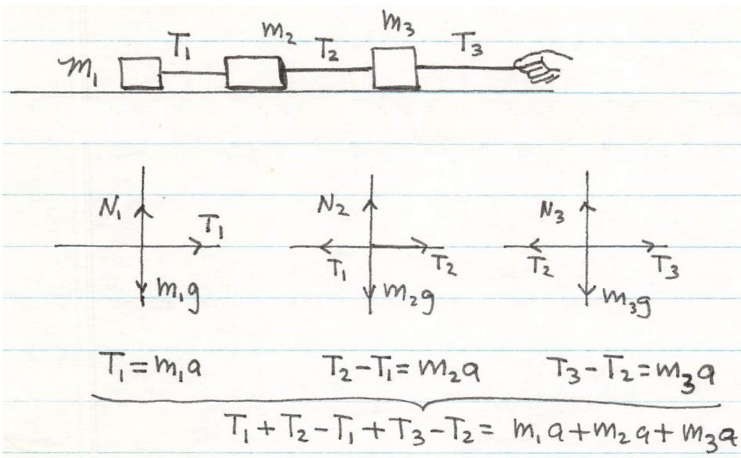
3. The length: $r = L\sin\theta$ => $L = \dfrac{r}{\sin\theta} = \dfrac{1.2}{\sin 20.556°} = 3.418 = 3.4\ \text{m}$.

E10. Railroad Cars. A leading car is pulling three railroad cars. You are seeing the last car in the foreground. Label the car closest to you 1. The next car is then 2, the next 3, and the fourth and final one is the lead car.

The weight of cars 1, 2, and 3 are 1.0×10^4 N, 2.0×10^4 N, and 3.0×10^4 N respectively, based on what is in them. The lead car is pulling on car 3 with a force $T_3 = 6.0 \times 10^3$ N. A force of 10^4 N is close to 1 US ton.

Let T_1 be the tension force in the connector between car 1 and car 2. Let T_2 be the tension force between car 2 and car 3. Find T_1, T_2, and the acceleration a in m/s^2. Neglect any friction.

Train in England, Courtesy Phil Sangwell, flickr, [Creative Commons Attribution]



Solution. A sketch is at the right where we replaced the lead car with a magic hand.

The equations are given at the bottom of each force diagram. We really do not need the vertical equations but it is best to always include all the forces acting on each body.

The application of Newton's Third Law gives us opposite action-reaction forces. This feature makes the algebra easy. We can add all of the following three equations that come from the force diagrams.

$$T_1 = m_1 a \qquad T_2 - T_1 = m_2 a \qquad T_3 - T_2 = m_3 a$$

$$T_1 + (T_2 - T_1) + (T_3 - T_2) = m_1 a + m_2 a + m_3 a$$

$$T_1 + T_2 - T_1 + T_3 - T_2 = (m_1 + m_2 + m_3)a$$

$$T_3 = (m_1 + m_2 + m_3)a$$

Given: $T_3 = 6.0 \times 10^3 \text{ N} \quad m_1 g = 1.0 \times 10^4 \text{ N} \quad m_2 g = 2.0 \times 10^4 \text{ N} \quad m_3 g = 3.0 \times 10^4 \text{ N}$

$$T_3 = (m_1 + m_2 + m_3)a \quad \Rightarrow \quad T_3 = (m_1 g + m_2 g + m_3 g)\frac{a}{g}$$

$$6.0 \times 10^3 \text{ N} = (1.0 \times 10^4 \text{ N} + 2.0 \times 10^4 \text{ N} + 3.0 \times 10^4 \text{ N})\frac{a}{g}$$

$$6.0 \times 10^3 \text{ N} = (6.0 \times 10^4 \text{ N})\frac{a}{g}$$

$$\frac{a}{g} = \frac{6.0 \times 10^3 \text{ N}}{6.0 \times 10^4 \text{ N}} = \frac{1}{10}$$

$$\boxed{\frac{a}{g} = \frac{1}{10}}$$

$$a = 0.1g = 0.98\,\frac{\mathrm{m}}{\mathrm{s}^2} \quad \Rightarrow \quad \boxed{a = 0.98\,\frac{m}{s^2}}$$

Summary: $T_1 = m_1 a$ $\quad T_2 - T_1 = m_2 a$ $\quad T_3 - T_2 = m_3 a$ $\quad \frac{a}{g} = \frac{1}{10}$

$$m_1 g = 1.0\times10^4\ \mathrm{N} \qquad m_2 g = 2.0\times10^4\ \mathrm{N} \qquad m_3 g = 3.0\times10^4\ \mathrm{N}$$

We can first solve for T_1.

$$T_1 = m_1 a = m_1 g\,\frac{a}{g} = 1.0\times10^4\ \mathrm{N}\cdot\frac{a}{g} = 1.0\times10^4\ \mathrm{N}\cdot\frac{1}{10} = 1.0\times10^3\ \mathrm{N}$$

$$\boxed{T_1 = 1.0\times10^3\ \mathrm{N}}$$

Next we go for T_2.

$$T_2 - T_1 = m_2 a$$

$$T_2 = T_1 + m_2 a$$

$$T_2 = T_1 + m_2 g\,\frac{a}{g}$$

$$T_2 = 1.0\times10^3\ \mathrm{N} + 2.0\times10^4\ \mathrm{N}\cdot\frac{1}{10}$$

$$T_2 = 1.0\times10^3\ \mathrm{N} + 2.0\times10^3\ \mathrm{N}$$

$$\boxed{T_2 = 3.0\times10^3\ \mathrm{N}}$$