A1. Physical Quantities. A physical quantity is a property that we can measure. Examples are your height, age, and weight. Each physical quantity has a number followed by a unit. Take height for example. In the photo below an employee at NASA is measuring the viewing height of another NASA employee, Lindsay Rodriguez, who is in a space suit.

Lindsay Rodriguez (c. 2005)
Courtesy NASA

"Photographic documentation of a Lander Ascent Stage Mock-up Suit Evaluation, with suited subject Lindsay Rodriquez. View of an unnamed employee measuring the viewing height of Lindsay Rodriquez, in an Advanced Crew Escape Suit (ACES) suit." NASA on The Commons, Flickr

Another physical quantity is age, e.g., someone might be 16. We would say the age is 16 years, with 16 being the number and years being the unit.

Author with Grandson

It is always good practice to include the units and not make assumptions. In everyday life, if someone in college said they were 20, you would know it is years. How about my grandson at the left? If I said he was 5, would you think 5 days, 5 weeks, 5 months, or 5 years? You would probably figure it out, but this guessing in general is very bad.

Always give the units in your final answers to assignments. Often, we will work out problems with units included as we go. You shall see such examples.

When the units are not included, there are chances of confusion and serious errors. We give the extremely expensive NASA error next, where a $328 million mission failed due to an assumption of incorrect units.

The Mars Climate Orbiter was sent to Mars back in the late 1990s. The mission "was unsuccessful due to a navigation error caused by a failure to translate English units to metric." Jet Propulsion Laboratory, Caltech. "Last contact with the spacecraft was on September 23, 1999, 9 months after launch, and an investigation found that the spacecraft burned up in Mars' atmosphere." Jet Propulsion Laboratory, Caltech.

Artist Conception of the Mars Climate Orbiter. Courtesy NASA.

The Author's Wife Weighing Herself

Our third physical quantity, weight, is illustrated at the left. A typical scale in the USA has units of pounds. We will learn about the metric unit newtons later. Weight is often reported in the medical profession as kilograms. The conversion they use is 1 kilogram is equivalent to 2.2 pounds. But the unit kilogram is technically not a weight. The unit kilogram refers to mass in the official metric system.

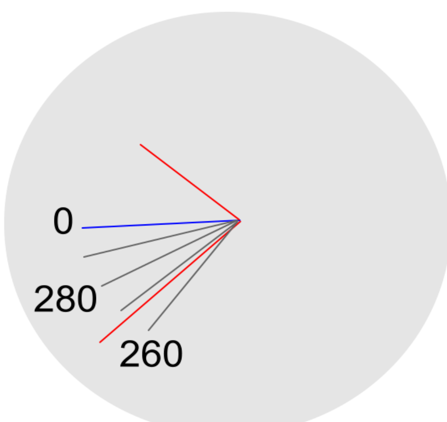
Mass and weight are different. As an example, if a suitcase weighs 60 pounds on Earth, that same suitcase would weigh 10 pounds on the Moon. But the mass, a measure of the amount of stuff, is the same both on the Earth and on the Moon.

We can make the medical use of the unit kilogram for weight please physicists by saying kilogram-weight. One kilogram-weight is the weight of a one-kilogram mass on Earth, which is a weight of 2.2 pounds.

The subtlety between mass and weight will become more clear when we treat mass and weight in greater detail later in our course. For now, just note that mass is a measure of how much stuff you have and that does not change if we put you on the Moon (except for a needed spacesuit). Weight, on the other hand, is the force that pulls you down to the Earth, which force we can measure if you stand on a scale. But if you stand on the same scale on the Moon, you will get 1/6 the value since the gravitational pull of the Moon on you is 1/6 of what the Earth does.

Female Cat (named Oshejoy) on Scale
Courtesy Alasam, Flickr
[Creative Commons (CC BY-NC-ND 2.0)]

Notice that Oshejoy is blocking the actual reading on the scale. She is hiding her weight. But physicists love challenges and solving problems. The next figure indicates how we might arrive at Oshejoy's weight using two angles that are equal. We start with making an angle using the needle and a line from the center to zero.



We then construct an equal angle on the other side of zero. From 0 to close to 270 lb, the span in the lower angle, corresponds to about 34 lb.

But wait! That weight is too high for a cat. Since we cannot see the entire scale, we surmise that the owner is pushing down on the scale at the edge. If you google Burmese cat, or just cat in general, you will find that a typical weight for a cat is about 10 lb.

In physics we always need to assess our answers. Always ask: Is the answer reasonable?

We should always be cautious. A colleague of mine would often say "Trust no one." Even double check yourself! In our next section we give a general guide in solving problems, even those outside physics.

A2. Guide in Problem Solving. When I was in school, I was taught in physics to start with the question and "the given." Write down what is given. If a diagram is included in the problem, sketch it first and list the given info nest to it. Then you pull out a formula or two and solve. You then check your work. Is your answer reasonable? Finally, you report your answer with proper units and significant figures. We treat significant figures in the next section. Here, our focus is on the general approach to problem solving.

Many years ago, the school I was teaching at, UNC-Asheville, was due for its usual 10-year reaccreditation and the official external accreditation committee gave us a homework assignment that would take several years to implement. We had to come up with a scheme that would be applicable across all disciplines in all courses. Our solution was summarized by Inquiry-ARC, where the letters stand for I = Inquiry, A = Apply, R = Reflect, and C = Communicate. Surprisingly to me, this approach to problem solving agreed with what I had learned in physics courses decades earlier.

The first step in any physics problem is to ask a question – the inquiry part. Then, the "arc" of the procedure includes the apply, reflect, and communicate steps. You apply physics and math to solve the problem – the apply stage. The next phase is to ask yourself if the answer is reasonable. It is so easy to punch in numbers incorrectly in a calculator and get strange results. If you are trying to figure out how long it takes a car to go 100 miles at 50 miles per hour and if your answer is 5000 hours, you did something wrong. So you go back and check things. In this case, we accidentally multiplied the numbers instead of dividing 100 by 50 to get the correct answer of 2 hours. Finally, you communicate your answer with the proper unit or units after the number and the proper amount of significant figures, which we will explain in the next section.

Though my school is now in a later phase of re-accreditation and the I-ARC model has been retired, I still like to use it. A logo of the model is at the left.

Here is how you can used the model to outline the process of invention and business enterprises. I will use Edwin Land (1909-1991) as the example. Legend has it that his daughter asked him why she couldn't see the picture right away after he took her photo in the early 1940s. Back in those days you had to send your film to Kodak, a camera company that developed it into a photo and then mailed it back to you with the negatives. So, according to legend, the daughter posed the "inquiry" – the question.

Land later in life said he always starts with a fantasy; you imagine something as perfect. So he imagines putting the chemical lab to develop the film in the camera and getting the picture in one minute. On a long walk in Santa Fe he came up with the essential solution over about 3 hours. But it took a few years to work out the details, applying the necessary science and testing. You apply science to build a prototype – the apply state. The testing is the reflecting stage. But here you go back and forth many times: prototype – test, next prototype – test, and so on. Finally you arrive at a suitable product and then sell it! To be successful in selling, you have to communicate to the public. The table below in the invention and business columns shows these steps relating to entrepreneurship.

I-ARC Model	Physics	Invention	Business	Pianist
Inquire	Question	Fantasy	Vision – CEO	"Fantasy" Performance
Apply	Laws/Math	Prototype	Research & Development	Practice at Home
Reflect	Check Answer	Test		Test Playing with a Teacher
Communicate	Proper Units	Sell It	Marketing	Recital

I also threw in an example of the process of becoming a pianist. You can replace this goal with becoming a basketball player or other profession. You imagine yourself on the stage giving the perfect performance or on the court playing basketball like a pro. To achieve the goal, you practice for hours, reflecting and self-correcting as you go. There are stories of the legendary basketball pro Michael Jordan, who played college ball in my state at the University of North Carolina, practicing for hours and hours. A good teacher or coach helps keep you on track, saving you lots of time. All this leads to a recital or basketball game with an audience. Then the process repeats with a harder musical composition or more challenging sports opponents.

For the business example, the Chief Executive Officer, the CEO, has the vision and starts the company. Land's company was the Polaroid Corporation, first making sunglasses, then cameras that developed the film on the spot – the instant camera. Here is the description of invention in Land's words:

> "You always start with a fantasy. Part of the fantasy technique is to visualize something as perfect. Then with the experiments you work back from the fantasy to reality, hacking away at the components." (Reference Edwin Herbert Land, May 7, 1909 - March 1, 1991 by Victor K. McElheny, National Academy of Sciences).

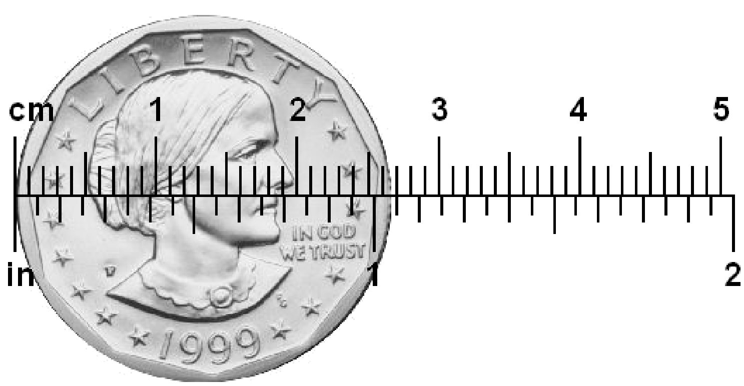
Steve Jobs (2010) and iPhone
Courtesy Matthew Yohe
at en.wikipedia

Research and development is "hacking away at the components." I like to take the time to include a section like this one in our course since physics is related to engineering and making products to sell. Inventors, engineers, and entrepreneurs can be inspirational to physics students. The making of prototypes and testing falls under research and development in a company. The same goes for software development – make a version and then iron out the bugs. Finally, communicating the product to the public is handled by the marketing division.

Land perfected the grand announcement of a new invention, where he would show up with the product on stage and invite the media. Steve Jobs (1955-2011) was so impressed by Land's approach that Jobs incorporated this procedure in the announcement of his Apple products.

A3. Significant Figures. There are different approaches to understanding significant figures. Many authors start out by giving rules. That approach is solid and acceptable. You can easily google significant figures and find such rules. My approach will be to discuss rounding off measurements and arriving at the rules through understanding. In this sense, we will derive the rules.

You should know that I am a theoretical physicist and that I like to derive everything myself. My approach to physics also incorporates interdisciplinary connections. I do not want to duplicate the many classic and excellent works on introductory physics. So for traditional approaches, I direct you to pretty much any introductory text in physics. But beware, these texts tend to be 1000 pages long. One advantage of our informal approach, is that our text is much shorter, even with interdisciplinarity present. Nevertheless, I highly recommend that you consult traditional texts also. Those books usually include Physics I and II, two semesters. Mechanics is the main focus of the first semester, while electricity and magnetism take center stage for the second semester. Our text here cover the first semester.



The concept of significant figures emerges from making measurements. So we star with a measurement. At the left we are measuring the diameter of the US Susan B. Anthony silver dollar coin. The coin was minted from 1979 to 1981, and then again in 1999.

There is some uncertainty in any measurement. In this case, we cannot precisely place the ruler on the edge and read the other end exactly. From the figure, we can say that the diameter is between 2.6 centimeters (2.6 cm) and 2.7 centimeters (2.7 cm). The coin's right edge looks like it is about midway between 2.6 and 2.7 cm. We might say 2.65 cm or 2.66 cm. So what should we report? Publications in physics often give a plus or minus, such as $d = 2.65 \pm 0.01\text{cm}$. Or, we can report 2.65 cm. Since there are 10 millimeters in 1 centimeter, we can also write $d = 26.5\text{mm}$. If we look up the specifications for this coin, we actually find $d = 26.5\text{mm}$. We say that we have three significant figures, even though there can be a little uncertainty in the last digit at the far right.

But if we do a gross round off to 30 mm, then we do not consider that “0” significant. After all, we could have 26, 28, 31 or 33. All of these numbers, and even a few more, round off to 30. So zeros at the far right of whole numbers do not count as being significant. Therefore, the following numbers all have only one significant figure: 3, 30, 300, and 3000. But a zero in between two nonzero numbers is significant. The number 303 has three significant figures. There is no round off for that zero in the middle. That zero clearly means no tens. The 303 indicates 3 hundreds, no tens, and 3 units.

If we know something is 300 where there is no gross rounding off, then we need to place a decimal point after the number. Writing 300. with that decimal means the zeros are now significant. We have three significant figures. There is no gross rounding off. The number 300. is like writing 301 or 302, three significant figures in each case.

Now consider 4 pennies. Here we have only one significant figure. Suppose we write it in terms of dollars? Then we have $0.04. The two zeros to the left of the 4 do not add any more information or precision to our value. Therefore, these zeros to the left of the 4 are not significant. More significant figures would mean more precision, e.g., going from 4 to 4.1 indicates more detail in a measurement. But with the leading left zeros, there is no new information.

What about a zero after a decimal point? If we start with 1.5 and then make it 1.50, we have added more precision. This zero is significant because we are intentionally indicating more information. Similarly, 1.500 has four significant figures. In summary, we have derived the following rules.

1. Each nonzero single-digit number (1, 2, 3, 4, 5, 6, 7, 8, 9) gives us one significant figure.
2. Any zero falling between two nonzero numbers is significant.
3. Zeros to the right of a whole number are not significant.
4. Leading zeros to the left are not significant.
5. A decimal point after a zero indicates that the zeros immediately to the left are significant.
6. A zero placed after a number on the decimal side is significant.

When we do calculations, we report the final answer with a number of significant figures corresponding to our weakest or most imprecise measurement. Consider a case where a vehicle travels at 10.1 meters per second (10.1 m/s) for 1.2 seconds (1.2 s). The distance traveled is

$$d = 10.1 \frac{\text{m}}{\text{s}} \cdot 1.2 \text{ s} = 12.12 \text{ m} .$$

Since our most imprecise value in the given information has 2 significant figures, i.e., the 1.2 s, we should report our final answer to 2 significant figures, giving

$$d = 12.12 \text{ m} = 12 \text{ m}.$$

Note that in physics and engineering we write $12.12 \text{ m} = 12 \text{ m}$, while in a math class they might freak out. You could ask, should we write $12.12 \text{ m} \approx 12 \text{ m}$? No. The practice is to write $12.12 \text{ m} = 12 \text{ m}$. We can interpret this equality by saying that 12.12 is equal or equivalent to 12 when we want two significant figures.

At times we will encounter a number that is precisely known, such as 2 channels for a stereo system. The 2 in this case is exactly 2, i.e., 2.0000000000000…, with an infinite amount of significant figures. So we might add a 7th rule.

7. An exact number such as 2 = 2.00000… has an infinite amount of significant figures.

Later in this chapter we will set up a calculation for one channel in a stereo system and we can appropriately report the answer to a certain amount of significant figures. To apply our result to two channels, we can simply include a factor of 2 in the calculation without reducing the number of significant figures.

One final comment. When working out numerical calculations, I keep at least two extra significant figures during intermediate steps even though I will discard these extra figures at the end of the calculation, rounding appropriately. I do not want to round off in the middle of the calculation, but save that for the end. Otherwise, I might "round off my answer away"!

I have found that chemistry professors are extremely strict with reporting the final answer to the amount of significant figures corresponding to the value used with the least amount of significant figures. When I taught engineering courses for seven years, I noticed that engineers often include an extra significant figure in the final answer. In our previous example with the 12.12 m, a chemist would report 12 m, while an engineer might give 12.1 m in a solution to a homework problem. For physics, we can use either the strict chemist rule or include an extra significant figure like the engineers.

A4. Derived Units from the Big Three. The three basic units in physics correspond to distance, time, and mass. One standard set of units for these in the Metric System is the meter, second, and kilogram. We abbreviate this standard as the MKS system, for meter, kilogram, and second. Another standard is the cgs system, which stands for centimeter, gram, and second. These systems are agreed-upon conventions that have become historical standards.

The point I want to emphasize here is that virtually all the units we use in our first introductory physics course can be derived as compound units from the three base units for length, time, and mass. A classic example is speed (or velocity v), which combines length and time. If you go 10 meters in 2 seconds, your speed is

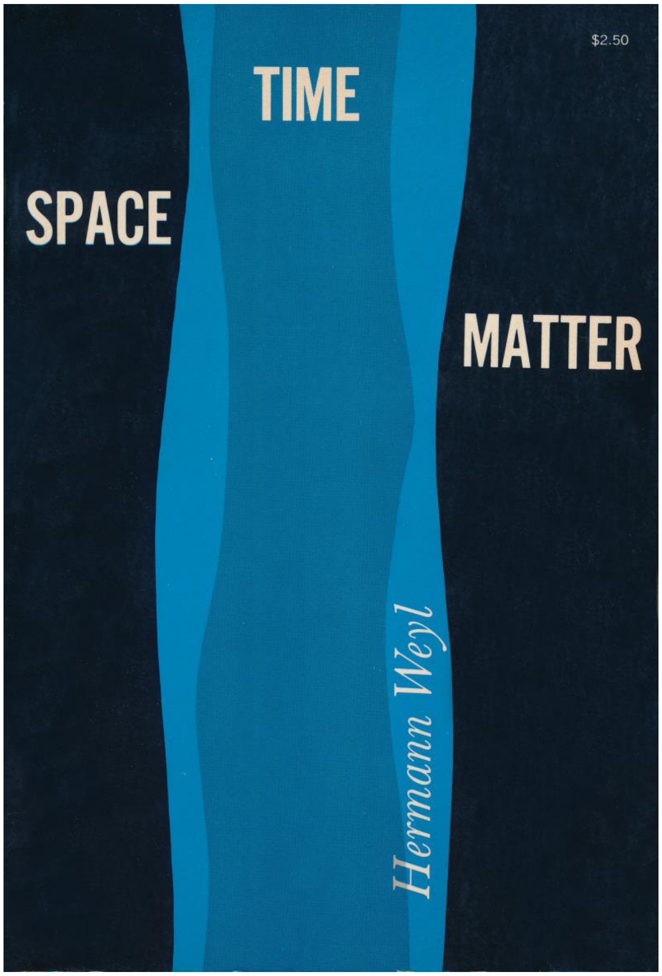
$$\text{speed} = v = \frac{10\ \text{meters}}{2\ \text{seconds}} = 5\,\frac{\text{m}}{\text{s}}\,.$$

We have a new physical quantity, speed, with units of length divided by time. We can write this result formally as

$$[\text{v}] = \frac{L}{T}\,,$$

which is read, the dimensions of v are equal to length (L) over time (T). Such a description of dimensions is called *dimensional analysis*.

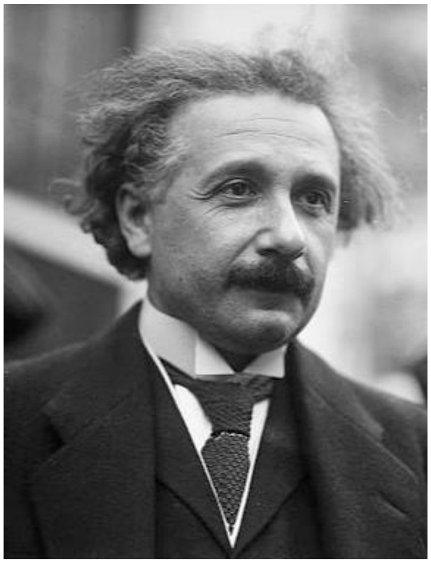
The "big three" – length, time, and mass – are a subset of the seven base quantities in the International System of Units (SI), which we often just call the Metric System. These seven physical quantities serve as the basis for deriving all other physical quantities. But watch how we can derive almost everything we need in our first introductory physics course from three: length, time, and mass.



Space, Time, and Matter
by Hermann Weyl
Dover Publications, Inc.
First English Edition, 1951,
translated from German
First German Edition, 1918
Purchased by the author c. 1970 for $2.50

The book at the left is an advanced book on space, time, and matter, originally written in German in 1918 by Hermann Weyl. Our key physical quantities are at play here: distance (the three dimensions making up space), time, and mass (matter).

The topic is Einstein's general theory of relativity, where space, time, and matter play a fundamental role. Einstein gave us a sophisticated edifice that incorporates these three concepts. In a nutshell, the fabric of spacetime is shaped by the presence of matter (and energy).



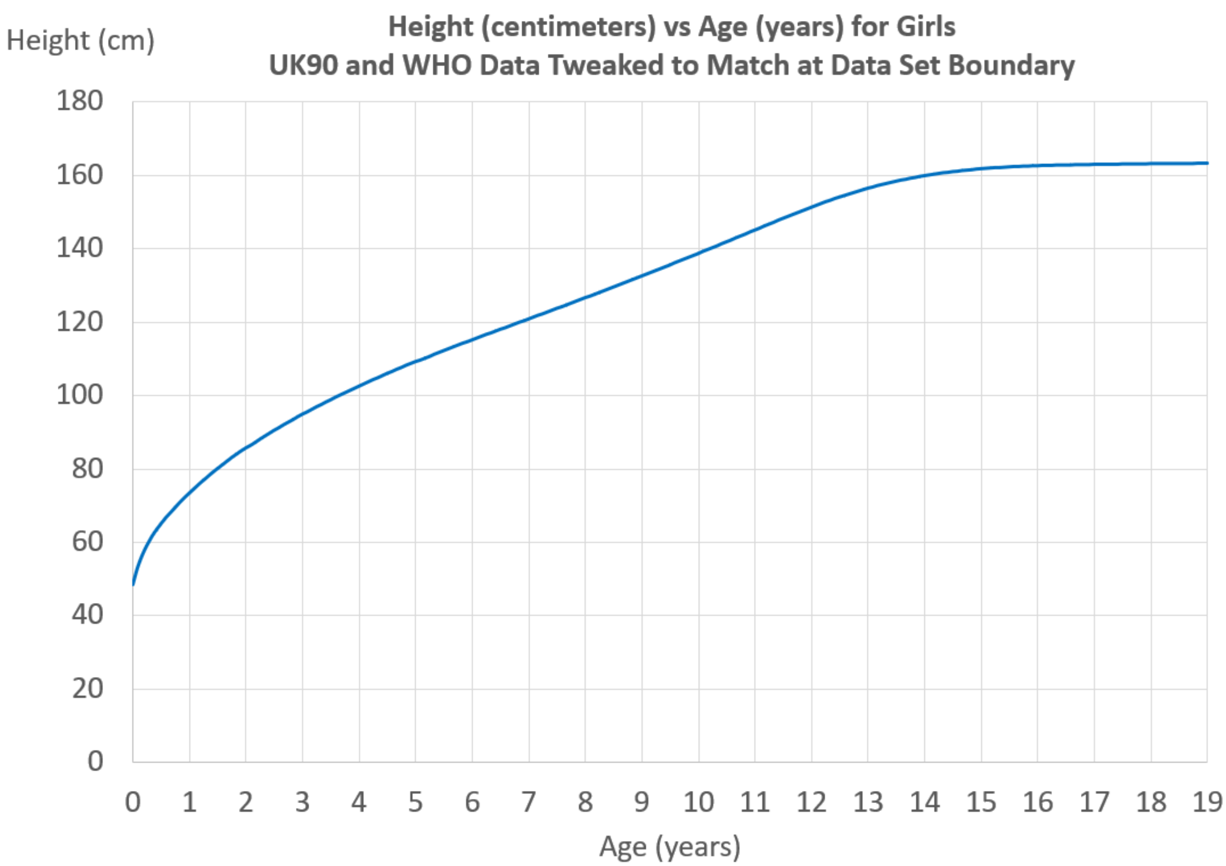
Albert Einstein (1879-1955)
Detail from a photograph by Harris & Ewing

Credit Line: Library of Congress, Prints & Photographs Division, photograph by Harris & Ewing, [reproduction number, e.g., LC-USZ62-123456]

We will not do much with Einstein in our introductory physics course as our focus will be on Newtonian physics, the physics of Isaac Newton (1643-1727).

Notice that when we mention "theory" in physics, we use the term in the sense of music theory, a verified body of knowledge through experiment, rather than hypothesis.

Two of our base quantities appear in the following graph, where we plot the height of females against time. The units for the height are centimeters (cm) and the units for time are years (y). The ages plotted are from birth, 0 y old, up to 19 years of age. The value are average values taken from data gathered by the United Kingdom (UK) and the World Health Organization (WHO). The data was tweaked ever so slightly so that the data sets could be joined seamlessly.



A derived physical quantity is the growth rate. Look at the height of 120 cm at 7 years of age. If you then fast forward to 10 years of age, the height is 140 cm. The climb of the graph over these 3 years is a fairly straight upward slope. The slope of the line is the rise over the run. The slope gives the growth rate, which is a velocity – though a very, very slow one.

$$\text{growth rate} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{140\text{ cm} - 120\text{ cm}}{10\text{ y} - 7\text{ y}} = \frac{20}{3}\frac{\text{cm}}{y} = 6.67\frac{\text{cm}}{y} = 7\frac{\text{cm}}{y}$$

A5. The Three Types of Physical Quantities.

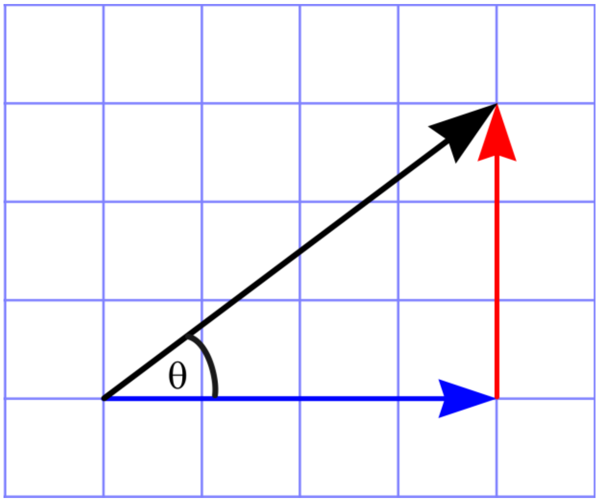
1. The Scalar. A *scalar* is a quantity that has a magnitude. You might wonder what else could a quantity have. You shall see shortly. For now, we just consider physical quantities with magnitudes. An example is temperature. You have a number and a unit such as 20 °C or 68 °F. The °C stands for degrees Celsius, while °F indicates degrees Fahrenheit. In the old days, the Celsius scale was known as the centigrade scale and one would have said in those days 20 degrees centigrade for 20 °C. But in 1948, they renamed the scale Celsius in honor of the Swedish scientist Anders Celsius (1701-1744), who was a professor of astronomy. But he also did work in physics, mathematics, meteorology, and geology.

Another example of a scalar is speed with its units of distance/time. But we can promote the speed to a new physical quantity, beyond scalar, one that also has direction. In such as case we can say that something goes 80 km/h to the east. This promotion takes us to the vector.

2. The Vector. A *vector* has magnitude and direction. The magnitude part is a scalar, such as 80 km/h. When you add the direction in there, you have a vector. We represent a vector as a line with an arrow that points in the proper direction. The length of the vector is a measure of the magnitude. When we promote speed to a vector, we use the term *velocity*.

Distance, such as 5 meters, is a scalar. But if you say walk 5 meters north, now you have a vector. We use the term *displacement* for a distance with a direction. The table below summarizes these important concepts.

Physical Quantities	
Scalar	**Vector**
distance	displacement
speed	velocity



In the left figure we take 4 paces to the east. This segment (horizontal and blue) is our first displacement vector. Then we proceed north for 3 paces (vertical and red). The net result is a distance of 5 paces with the angle shown. We have used the Pythagorean theorem to determine the length of the sum vector, which we call the *resultant*.

Let r be the magnitude of the resultant. The magnitude is

$$r = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5 .$$

The angle theta is found from $\tan\theta = 3/4$. The angle is $\theta = \tan^{-1}(3/4) = 37°$. The resultant is in the northeastern direction at a bearing of 37° from the east.

Note that we add vectors by joining the tail of one to the end of the other. You can move vectors around as long as you do not stretch them or rotate them. With such careful movement, the length and direction are preserved and therefore, the vector does not change. This feature is important for adding vectors. Walking along the vectors in our previous figure indicates that this method of addition is the correct one from a common-sense perspective. Let the symbol $\hat{i}$ represent one step to the east and j represent one step to the north. Then we can write for the sum vector,

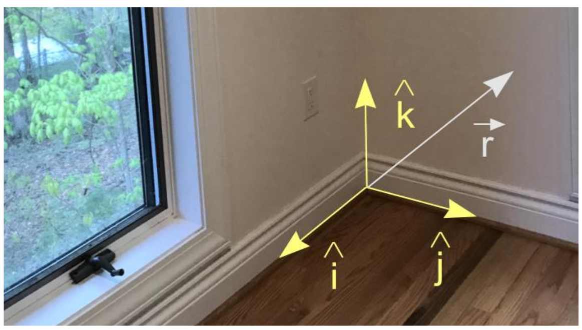
$$\vec{r} = 4\hat{i} + 3j .$$

The above expression is vector notation. Note the little arrow over the r. In three dimensions, we can write a vector as

$$\vec{r} = x\hat{i} + yj + zk\ ,$$

Where $\hat{i}$, j, and k are the unit vectors along each of the three dimensions: length, width, and height.

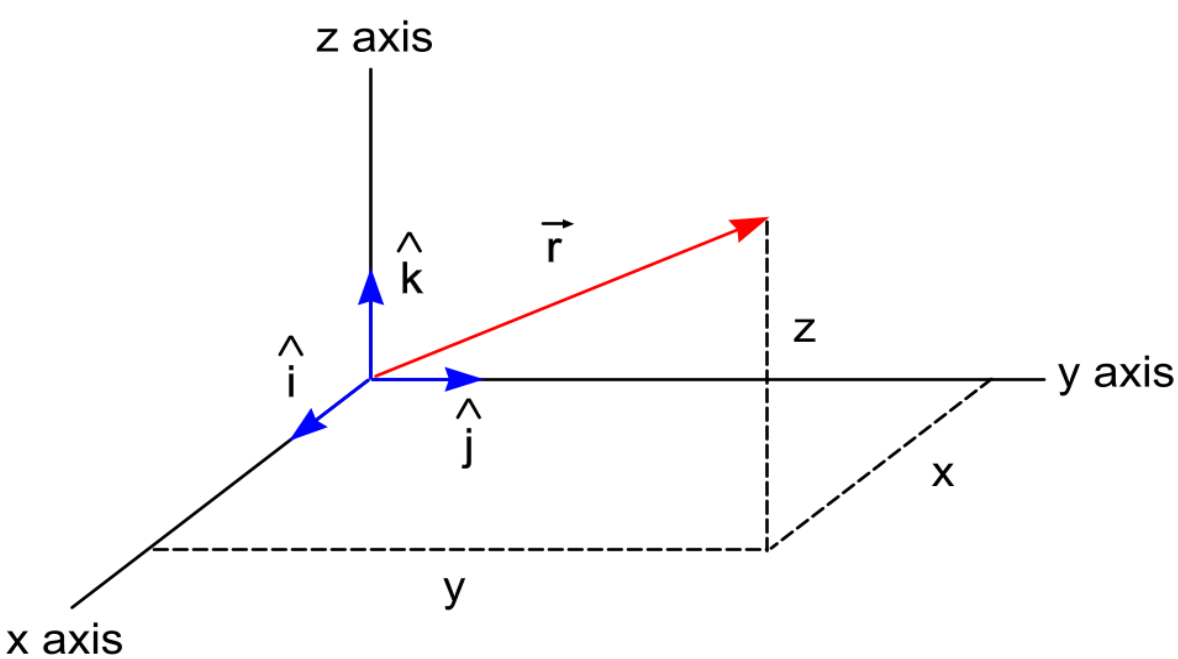
See figures above and below for the unit vectors in each of the three spatial dimensions. Each point in the room has 3 coordinates: how many paces along the $\hat{i}$ direction (which we can call x), how many paces along the j direction (which we can call y), and how many paces along the k direction (which we can call z). The each point in the room can be represent by coordinates (x, y, x). We can also draw a displacement vector from the reference corner to the point in the room. This displacement vector is then



$$\vec{r} = x\hat{i} + yj + zk\ .$$

Some texts use bold for a vector instead of the cute arrow on top of a letter.

r = $\vec{r}$



3. The Tensor. There's more? Yes. An applied force vector is shown in the figure. Imagine someone with a blunt tool pushing on the board in the direction of the applied force vector.

The board's orientation is designated by the white vector that points outward and perpendicular to the board surface. The applied force has three components, one for each of the three dimensions.

$$\vec{F} = F_x \hat{i} + F_y j + F_z k$$

But each of these three components can in general push the board inward, stretch the board downward, and stretch the board sideways. Therefore, there are nine possible stresses on the board. We can designate these as elements of a 3 x 3 listing of rows and columns, called a *matrix*.

$$\sigma_{kl} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

Such a physical quantity with some additional properties we will not go into is called a *tensor*. Specifically, this tensor is one of rank 2 because if has two subscript indices, indicated by k and l for row and column. There are tensors that can have more indices. Our course will not delve into tensor physical quantities. Scalars and vectors will keep us pretty busy. A vector can be thought of as a tensor of rank 1 and a scalar is a tensor of rank 0.

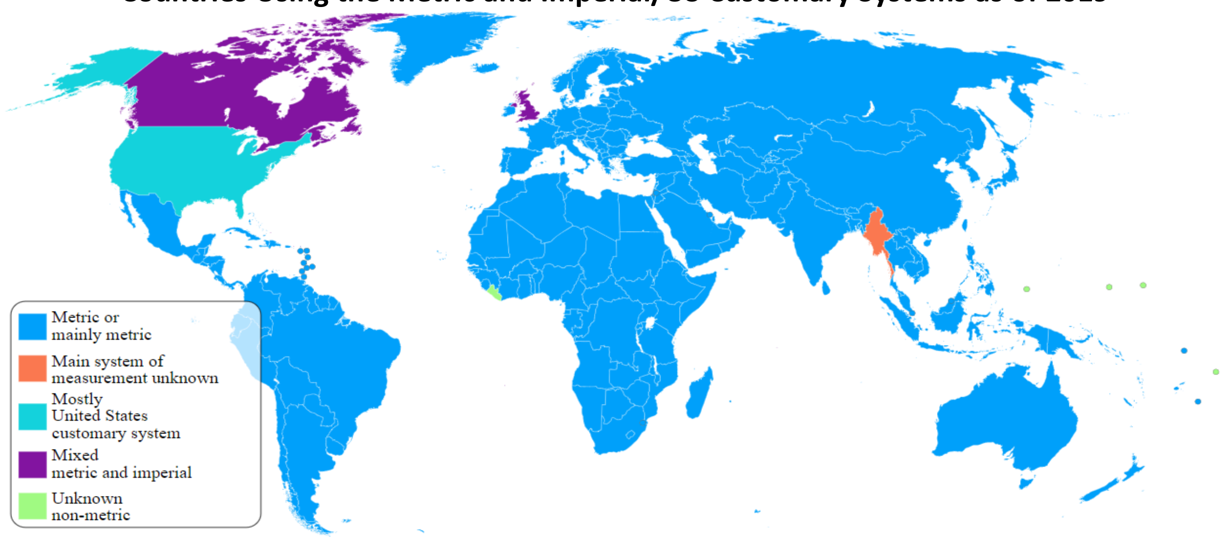
On a personal note, I fell in love with tensors through a study of Einstein's General Theory of Relativity (1915). The Einstein tensor field equations are shown below.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

The $G_{\mu\nu}$ piece is a tensor that describes the geometry of spacetime (four dimensions: 3 for space and 1 for time) due to the presence of matter and energy contained in the $T_{\mu\nu}$ tensor. The tensor $g_{\mu\nu}$ also describes aspects of the geometry of spacetime. The rest of the terms are constants. The constant Λ is the "infamous" or "famous" cosmological constant, G is Newton's gravitational constant (which we will cover in this course), and c is the speed of light.

A6. The Metric System. The Metric System is used throughout the world, with a major exception being the United States. The Metric System was born in the 1790s during the French Revolution and spread throughout the world in the 1800s. Before the advent of the Metric System, England was the first country to industrialize, starting as early as 1760. It began using its own system of units for specs in manufacturing machines. The units used in England are described as the British System or the Imperial System. Examples of lengths in this system are inches, feet, yards, miles. The United States developed its system called Customary Units in the early 1830s, derived from the British System. While one might argue that the highly successful industrial revolution in the US made it very costly to switch at that time, it is hard to justify that in the 21st century, the US still holds on to its old established system of units. However, scientific journals published in the United Kingdom and the United States have adhered mainly to the Metric System for some time now.

Countries Using the Metric and Imperial/US Customary Systems as of 2019



Courtesy Goren tek-en, Wikimedia, Creative Commons

Students in the US have a more difficult task learning physics since they are required to use the Metric System in their courses. However, the computer revolution has introduced many metric conventions as we shall see shortly. The Metric System is based on a prefix that stands for a power of ten such as 100, 1000, or 1/10. Below are three very common prefixes.

Metric Prefix	Abbreviation	Value	Sample Usage
centi	c	1/100	centimeter (cm)
milli	m	1/1000	millimeter (mm)
kilo	k	1000	kilometer (km)

Next we will consider common Metric prefixes used in computer technology, though be careful, the computer folks define a kilo as 1024, the nearest power of 2 closest to 1000.

Power of 2	0	1	2	3	4	5	6	7	8	9	10
Value	$2^0=1$	$2^1=2$	$2^2=4$	$2^3=8$	16	32	64	128	256	512	1024

Metric prefixes are more common due to computer hard drives. When I went to school, I had to learn most of these since we didn't have hard drives. The superscript with 10 means how many zeros follow the one. As an example, 10^3 indicates three zeros after the 1, which is 1000. This value is one thousand. This superscript is called powers-of-ten notation and it frees us from having to write down lots of zeros. A favorite of mine is 10^{100}, 1 followed by 100 zeros, which is the *googol*, but this number does not have a metric prefix. My best friend in grade school told me about the googol and associated *googolplex*. The *googolplex* is a 1 followed by a googol of zeros. Think about that! I finally understood it by an analogy with a thousand = 10^3 and considering a 1 followed by a thousand zeros.

Metric Prefix	Abbreviation	Description	Numerical Value
kilo	k	thousand	$1{,}000 = 10^3$
mega	M	million	10^6
giga	G	billion	10^9
tera	T	trillion	10^{12}

If you spell googol as Google you get the famous search engine. If you spell googolplex as Googleplex, you get the building below.

Googleplex Headquarters, Mountain View, California, USA

Courtesy The Pancake of Heaven!, Wikimedia, Creative Commons

The Metric System

Below is a listing the metric prefixes Courtesy Wikipedia. Don't memorize all of them. We will learn many throughout our course.

SI Prefixes Courtesy Wikipedia

Prefix		Base 10	Decimal	English word		Adoption[nb 1]
Name	Symbol			Short scale	Long scale	
yotta	Y	10^{24}	1000000000000000000000000	septillion	quadrillion	1991
zetta	Z	10^{21}	1000000000000000000000	sextillion	trilliard	1991
exa	E	10^{18}	1000000000000000000	quintillion	trillion	1975
peta	P	10^{15}	1000000000000000	quadrillion	billiard	1975
tera	T	10^{12}	1000000000000	trillion	billion	1960
giga	G	10^{9}	1000000000	billion	milliard	1960
mega	M	10^{6}	1000000	million		1873
kilo	k	10^{3}	1000	thousand		1795
hecto	h	10^{2}	100	hundred		1795
deca	da	10^{1}	10	ten		1795
		10^{0}	1	one		–
deci	d	10^{-1}	0.1	tenth		1795
centi	c	10^{-2}	0.01	hundredth		1795
milli	m	10^{-3}	0.001	thousandth		1795
micro	μ	10^{-6}	0.000001	millionth		1873
nano	n	10^{-9}	0.000000001	billionth	milliardth	1960
pico	p	10^{-12}	0.000000000001	trillionth	billionth	1960
femto	f	10^{-15}	0.000000000000001	quadrillionth	billiardth	1964
atto	a	10^{-18}	0.000000000000000001	quintillionth	trillionth	1964
zepto	z	10^{-21}	0.000000000000000000001	sextillionth	trilliardth	1991
yocto	y	10^{-24}	0.000000000000000000000001	septillionth	quadrillionth	1991

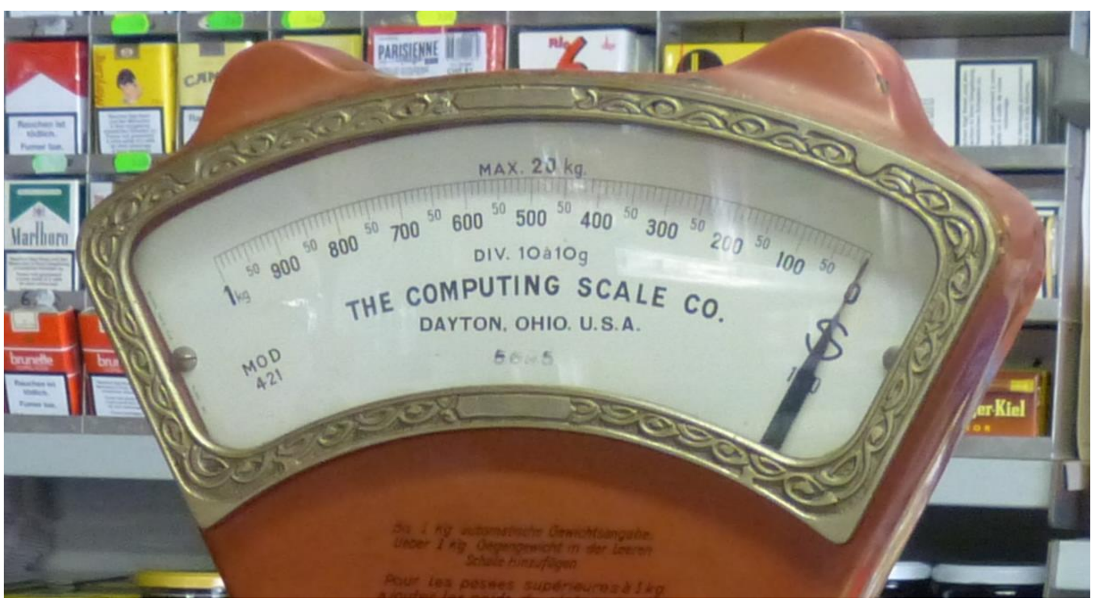
1. ^ Prefixes adopted before 1960 already existed before SI. The introduction of the CGS system was in 1873.

Unit of Mass: The gram

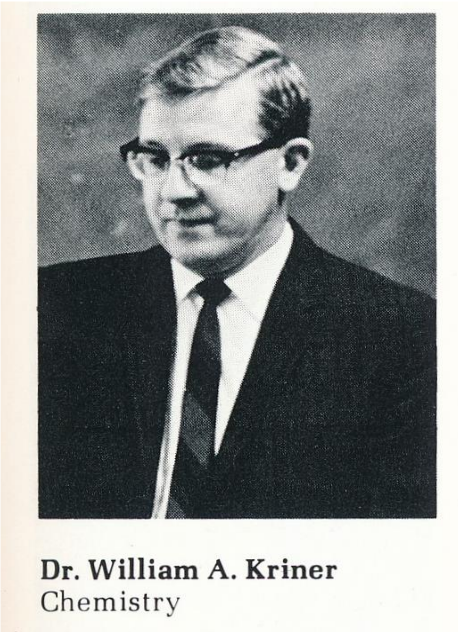
A US penny has a mass of 2.5 grams (2.5 g).
Courtesy United States Mint (Public Domain)

English: Antique Computing Scale Co. scale still in use in shop in Röthenbach, Switzerland
Courtesy GabrielleMerk, Wikimedia, Creative Commons


MAX. 20 kg
DIV. 10à10g
THE COMPUTING SCALE CO.
DAYTON, OHIO. U.S.A.
MOD 421
Automatische Wage
Bascule automatique
3.70
TOTAL
CHANGE
CASIO
AUSZEIT
nutella
Lipton
NESCAFÉ GOLD
Ovomaltine
original
THOMY
100%


MAX. 20 kg.
1kg 50 900 50 800 50 700 50 600 50 500 50 400 50 300 50 200 50 100 50 0
DIV. 10à10g
THE COMPUTING SCALE CO.
DAYTON, OHIO. U.S.A.
MOD 421
Marlboro
PARISIENNE

A7. Working with Units. 0. The Trick. My chemistry teacher in college taught me the trick with setting up units and watching them cancel. It was my freshman year in Fall 1968.



Dr. William A. Kriner, Chemistry Professor
St. Joseph's College (now University)
Philadelphia, Pennsylvania, USA
Photo St. Joseph's College Yearbook: *1972 Greatonian*

Here is a simple example and why it works. Consider converting 2 hours to minutes. You write down the 2 hours and then note that 1 hour = 60 minutes. Since 1 hour = 60 minutes, you can write this relation as

$$1 = \frac{60\ \text{min}}{1\ \text{h}} \quad \text{or} \quad 1 = \frac{1\ \text{h}}{60\ \text{min}}\ .$$

In each case, you have unity. Multiplying anything by unity does not intrinsically change anything, but it can convert the units.

$$2\ \text{hours} = 2\ \text{h} \cdot 1 = 2\ \text{h} \cdot \frac{60\ \text{min}}{1\ \text{h}} = 120\ \text{min}$$

1. Text. Question 1: Given one byte per character, estimate in megabytes (MB) what storage is necessary for a 400-page book (no photos).

We would like to estimate the storage capacity of a book with no illustrations or photos. We can start with how many characters are there in a line of text. If you count the characters in a line of this book you are reading now, you will find about 90 characters in a line. Remember, we are estimating. If you count the number of lines on a page, you get 40 to one significant figure. We are shooting for characters per book. The following dimensional analysis encapsulates our reasoning.

$$\frac{\text{char}}{\text{book}} = \frac{\text{char}}{\text{line}} \cdot \frac{\text{lines}}{\text{page}} \cdot \frac{\text{pages}}{\text{book}}$$

Using our values with a 400-page book, we have

$$\frac{\text{char}}{\text{book}} \Rightarrow \frac{90\ \text{char}}{\text{line}} \cdot \frac{40\ \text{lines}}{\text{page}} \cdot \frac{400\ \text{pages}}{\text{book}} = 1{,}440{,}000 \frac{\text{char}}{\text{book}}$$

Notice how several of the units cancel, leaving you with what you want. Since each character (char) is a byte, our book needs 1,440,000 bytes. What about kilobytes (kB)? Remember that 1 kilobyte is 1024 bytes since in computer science, a kilo = 1024.

$$\frac{\text{kilobytes}}{\text{book}} => 1,440,000 \cdot \frac{\text{bytes}}{\text{book}} \cdot \frac{1 \cdot \text{kilobyte}}{1024 \text{ bytes}} = 1406 \text{ kilobytes}$$

For Megabytes we proceed with the following conversion of units, using 1 Megabyte = 1024 kilobytes.

$$\frac{\text{Megabytes}}{\text{book}} => 1406 \frac{\text{kilobytes}}{\text{book}} \cdot \frac{1 \cdot \text{Megabyte}}{1024 \text{ kilobytes}} = 1.4 \text{ Megabytes} = 1.4 \text{MB}$$

The Library of Congress has about 40 million books. Assuming for now no pictures or photos, we will work up to the storage needed for 40 million books of text.

Number of Books (Text Only)	Storage
1	1.4 MB
1,000	1.4 GB
1,000,000	1.4 TB
10,000,000	14 TB
40,000,000	56 TB

2. Images. Question 2: What storage space in kB do you need for an image 800 x 640 pixels with 24-bit color depth per pixel and with a 1:10 jpg compression ratio? A byte is equal to 8 bits.

$$800 \times 640 \text{ pixels} \cdot \frac{24 \text{ bits}}{\text{pixel}} \cdot \frac{1 \text{ byte}}{8 \text{ bits}} \cdot \frac{1 \text{ kB}}{1024 \text{ bytes}} \cdot \frac{1}{10} = 150 \text{ kB}$$

To estimate a book with 40 images, we can add an additional 40 x 150 kB = 6000 kB = 6 MB. Note that for quick estimates we can take 1024 = 1000. Adding the 6 MB to the 1.4 MB found earlier and rounding up, we arrive at 10 MB per book.

Number of Books (with images)	Storage
1	10 MB
1,000	10 GB
1,000,000	10 TB
10,000,000	100 TB
40,000,000	400 TB

The next hard drive size would be a petabyte (PB), where 1 PB = 1024 TB. A petabyte drive should do it. By the way, the 24-bit color depth refers to 8 bits for each primary red, green, and blue. Each primary color has 8 bits = 1 byte of data.

3. Audio. Question 3: For CD sound, given 16-bits per stereo channel and a sampling rate of 40,000 1/s, estimate the storage necessary for 70 minutes of music.

$$\frac{16\text{ bits}}{\text{channel}} \cdot \frac{2\text{ channels}}{\text{sample}} \cdot 40{,}000\frac{\text{samples}}{\text{s}} \cdot \frac{1\text{byte}}{8\text{ bits}} \cdot \frac{1\text{kB}}{1024\text{ bytes}} \cdot \frac{1\text{MB}}{1024\text{ kB}} \cdot \frac{60\text{ s}}{\text{min}} \cdot 70\text{ min} = 640\text{ MB}$$

A CD with 650 MB of storage holds 74 minutes of music. We essentially obtained this result! We have roughly 650 MB/74 minutes = 8.8 MB/min. With mp3 compression, you can get this down to 1 MB/min, which is about 3 MB for a three-minute song. The compression will reduce the quality of the audio, but if you don't notice it hardly at all, that is a good thing.

Apple came out with a media player called the iPod in the early 2000s. If you had one with a 120 GB hard drive, you could store

$$120\text{ GB} \cdot \frac{1\text{ song}}{3\text{ MB}} \cdot \frac{1024\text{ MB}}{1\text{ GB}} = 41{,}000\text{ songs}$$

iPod 5th Generation, 2005
Courtesy Stahlkocher
Wikimedia, Creative Commons

Sometimes audio transfer of data is given as a bit rate in kilobits per second. Without compression,

$$\frac{650\text{ MB}}{74\text{ min}} \cdot \frac{1024\text{ kB}}{1\text{ MB}} \cdot \frac{8\text{ bits}}{1\text{ byte}} \cdot \frac{1\text{ min}}{60\text{ s}}$$

$$= 1200\ \frac{\text{kb}}{\text{s}} = 1200\text{ kb/s},$$

where little b stands for bits. If we kick in mp3 compression, we divide by 8 and find 150 kb/s.

Typical audio bitrates for streaming are 64 kb/s for low quality, 128 kb/s for DVD 720x480 pixel videos, and high quality 256 kb/s for 1920x1080 pixel video.

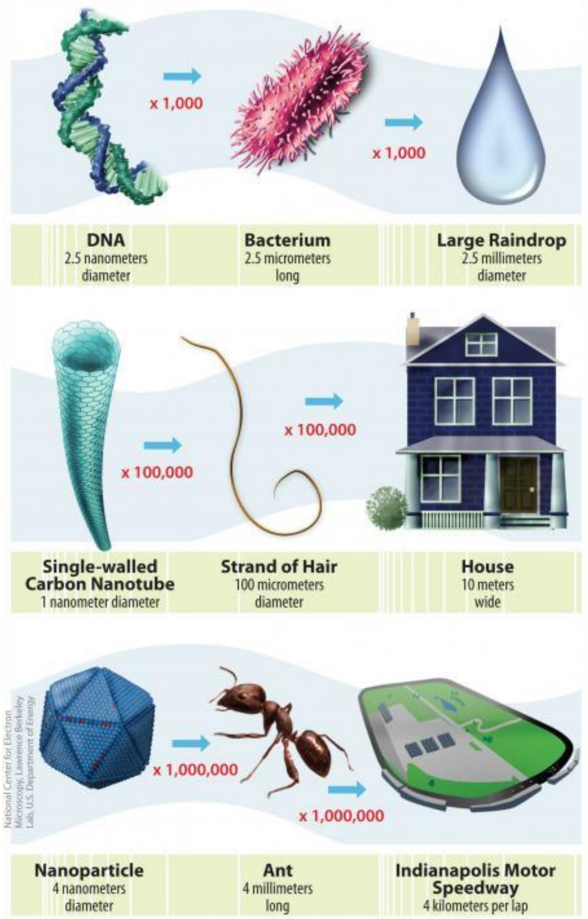
4. Video. Question 4: For a DVD video of 720 x 480 pixels per frame, 24-bit color depth per pixel, 30 frames per second, and a compression ratio of 40, find the GB storage space needed for a 2-hour movie. For the audio, use the result of the previous problem with a compression ratio of 4. First we do the video.

$$2\text{h}\cdot\frac{720\text{x}480\text{ pixels}}{\text{frame}}\cdot\frac{30\text{ frames}}{\text{s}}\cdot\frac{60\text{ s}}{\text{min}}\cdot\frac{60\text{ min}}{\text{h}}\cdot\frac{24\text{ bits}}{\text{pixel}}\cdot\frac{1\text{ byte}}{8\text{ bits}}\cdot\frac{1\text{ GB}}{(1024)^3\text{bytes}}\cdot\frac{1}{40}=5\text{ GB}$$

For audio: $2\text{h}\cdot\frac{640\text{ MB}}{70\text{ min}}\cdot\frac{60\text{ min}}{\text{h}}\cdot\frac{1}{4}\cdot\frac{1\text{ GB}}{1024\text{ MB}}=0.3\text{ GB}$.

The total storage is $5.3\text{ GB}=5\text{ GB}$ to one significant figure. With a terabyte hard drive, you can fit

$$1\text{ TB}\cdot\frac{1\text{ movie}}{5\text{ GB}}\cdot\frac{1024\text{ GB}}{\text{TB}}=200\text{ movies}.$$



The Scale of Things
Courtesy Nano.gov

With a futuristic petabyte hard drive, you can have 200,000 movies. Alexa, the Amazon assistant, says there are about 500,000 movies in existence today. Now we have been using 2 hours for movie. But most movies are shorter. So a petabyte comes close.

DNA can code up to about 100 petabytes. So if scientists of the future use nanotechnology to get this kind of data storage on a single disc, we should be able to easily include all the movies, all the music recordings, and all the books including those with figures.

The prefix nano in nanotechnology is a metric prefix equal to a billionth, i.e.,

$$\text{nano}=\text{n}=\frac{1}{1,000,000,000}=\frac{1}{10^9}=10^{-9},$$

where the minus sign in a power of ten means we take 1 over that number. Think of the minus sign in the exponent as "th" so we have billionth (-9 exponent) instead of billion (+9 exponent). Nanotechnology deals with sizes in the range of 1 nanometer to 100 nanometers, which we abbreviate as 1 nm to 100 nm. See the above figure for a sense of scale.

In this futuristic scenario, say everyone pays a monthly fee to have everything. Then you could get a small 100-petabyte hard drive disc to put in your pocket and you have everything covered. But more likely, everything will be streamed and you will not need much personal hard storage anyway on a local machine. Everything will be in the Cloud.

Cloud Photo by Doc from Airplane, 2013

HP-15C from the 1980s
Courtesy Hic et nunc
and Pittigrilli, Wikimedia

A8. Which Calculator Should I Use? I recommend a calculator that does not use parentheses. Such a calculator was the world's first pocket calculator, developed by Hewlett-Packard. It was the HP-35 and the year was 1972. HP calculators do not use parentheses, but instead, employ Reverse Polish Notation (RPN). HP calculators became very popular among scientists and engineers.

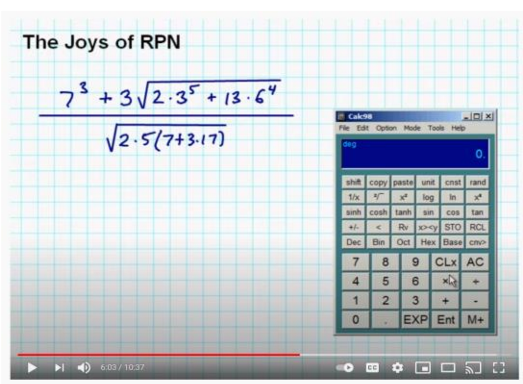
But algebraic notation, which uses parentheses, though conceptually easier for non-scientists, is not as good. Such is unfortunate since many educational systems push the algebraic calculators in lower grade levels. Below is an abstract from a research study that confirms RPN is superior to algebraic.

> “Two carefully matched calculators using different logic systems were compared using subjects experienced in each logic system. Reverse Polish Notation (RPN) proved superior to Algebraic Notation (AN) in all comparisons, although the differences in overall speed and accuracy were smaller than had previously been found. A classification scheme for user behavior was derived and used to show that the superiority of RPN was due to elimination of sequence errors and reduction of unnecessary steps in performing calculations.” D. M. Kasprzyk, C. G. Drury, and W. F. Bialis, “Human Behavior and Performance in Calculator Use with Algebraic and Reverse Polish Notation,” *Ergonomics* **22:9**, 1011-1019, 1979.

Here is a summary of my reasons for the RPN:

1. Hewlett-Packard, a great company, picked RPN for their calculators (first pocket one in 1972).
2. Entering numbers, then the operation, eliminates parentheses.
3. With RPN you learn about the stack, which is an important concept in computer science.
4. Research studies confirm the superiority of RPN: less keystrokes and less errors.

I use the HP-15C app on my iPhone 11 Pro Max, which app is identical to the actual HP-15C calculator of the 1980s. The app cost a few dollars. The size of my iPhone 11 Max Pro is about the actual size of the original HP-15C that I purchased in the early 1980s. It was $179, which was a lot of money back then and still is for a calculator. I have used the HP-15C for my entire career. If you have never used RPN, it is worth learning and giving RPN a chance. You can find YouTube videos on the topic.



Here is an example of such a YouTube video.

The Joys of RPN

The video compares algebraic and RPN logic. It even uses a calculator that lets you switch between algebraic and RPN modes. Check it out.