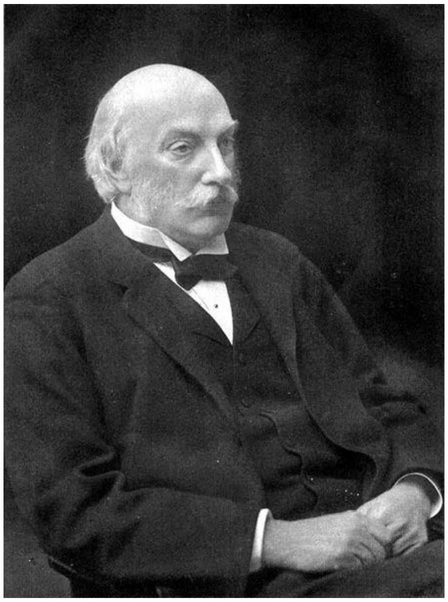
Modern Optics, Prof. Ruiz, UNCA **doctorphys.com**
Chapter P. Why is the Sky Blue?

P0. Lord Rayleigh



Lord Rayleigh

John William Strutt (1842 - 1919)
3rd Baron of Rayleigh
British Physicist

Both a Theoretical and Experimental Physicist
Nobel Prize in Physics 1904

"For his investigations of the densities of the most important gases and for his discovery of argon in connection with these studies." Nobel Committee

The first Nobel prize in Physics was awarded in 1901.

The Nobel in physics for the first seven years are listed below.

1901. Wilhelm Conrad Röntgen, “in recognition of the extraordinary services he has rendered by the discovery of the remarkable rays subsequently named after him”

1902. Hendrik Antoon Lorentz and Pieter Zeeman “in recognition of the extraordinary service they rendered by their researches into the influence of magnetism upon radiation phenomena”

1903. Antoine Henri Becquerel “in recognition of the extraordinary services he has rendered by his discovery of spontaneous radioactivity”

Pierre Curie and Marie Curie, née Sklodowska “in recognition of the extraordinary services they have rendered by their joint researches on the radiation phenomena discovered by Professor Henri Becquerel”

1904. Lord Rayleigh (John William Strutt) “for his investigations of the densities of the most important gases and for his discovery of argon in connection with these studies”

1905. Philipp Eduard Anton von Lenard “for his work on cathode rays”

1906. Joseph John Thomson “in recognition of the great merits of his theoretical and experimental investigations on the conduction of electricity by gases”

1907. Albert Abraham Michelson “for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid”

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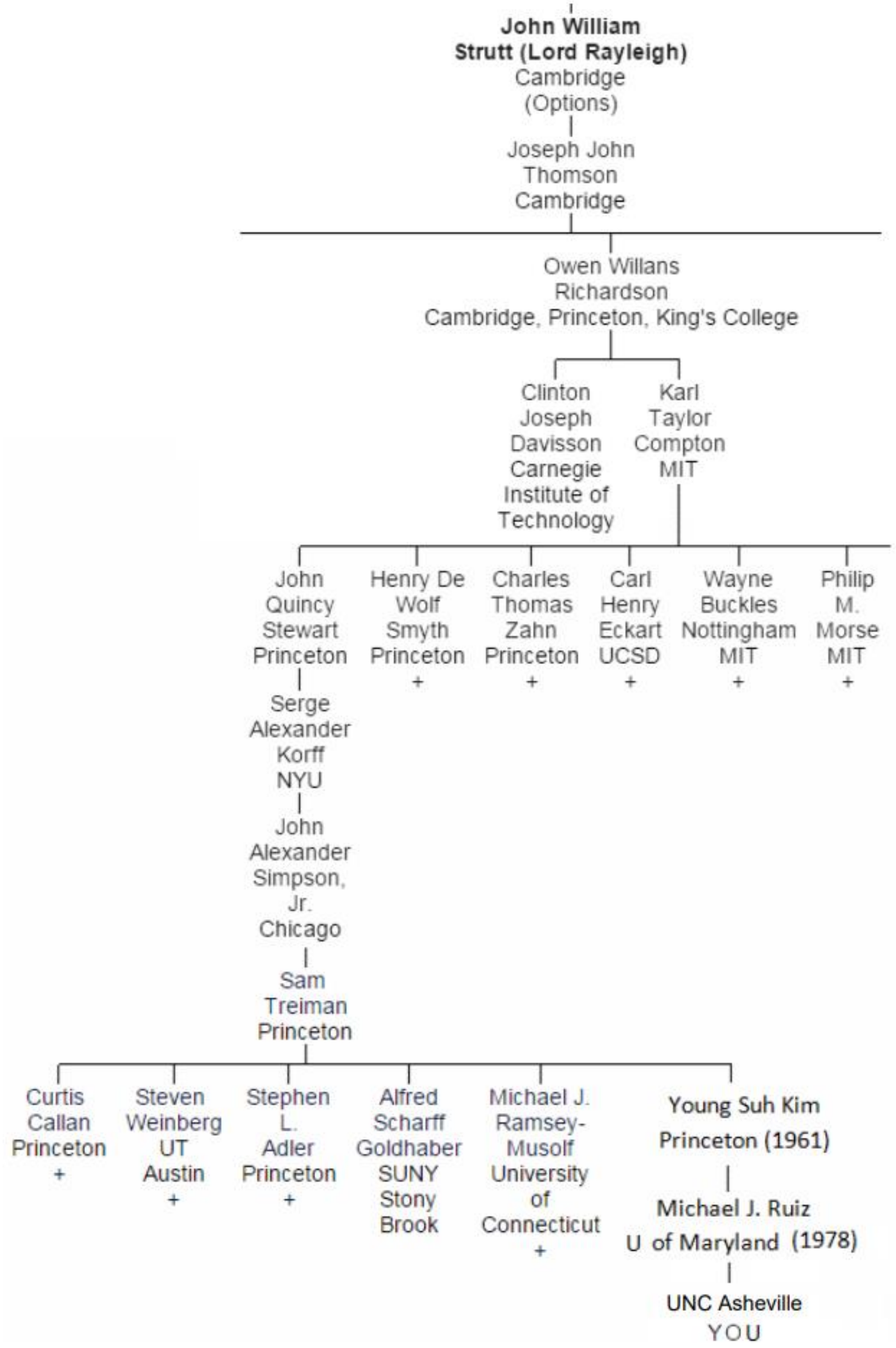
Your teaching line goes back to Lord Rayleigh (Nobel 1904). Note that directly under Rayleigh is his student J. J. Thomson (Nobel 1906), discoverer of the electron and winner of the 1906 Physics Nobel. And these go back to Newton and Galileo, as we have mentioned earlier.



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P1. Electric Polarization in Matter. (Not to be confused with the polarization of light.)

We need to return to the Maxwell equations in matter as we are interested in light interacting with particles in the atmosphere. This chapter is another overlap with Atmospheric Science or Meteorology.

From material in a previous chapter, we have the following.

Maxwell Equations In Vacuum	Maxwell Equations In Linear and Isotropic Matter
$\oiint \vec{E}\cdot\overrightarrow{dA}=\dfrac{Q}{\varepsilon_0}$	$\oiint \vec{E}\cdot\overrightarrow{dA}=\dfrac{Q_{free}}{\varepsilon}$
$\oiint \vec{B}\cdot\overrightarrow{dA}=0$	$\oiint \vec{B}\cdot\overrightarrow{dA}=0$
$\oint \vec{B}\cdot\overrightarrow{dl}=\mu_0 i+\mu_0\varepsilon_0\dfrac{d\Phi_E}{dt}$	$\oint \vec{B}\cdot\overrightarrow{dl}=\mu i_{free}+\mu\varepsilon\dfrac{d\Phi_E}{dt}$
$\oint \vec{E}\cdot\overrightarrow{dl}=-\dfrac{d\Phi_B}{dt}$	$\oint \vec{E}\cdot\overrightarrow{dl}=-\dfrac{d\Phi_B}{dt}$

The medium's permittivity $\varepsilon=\varepsilon_o(1+\chi_e)$, where χ_e is the electric susceptibility.

The medium's permeability $\mu=\mu_o(1+\chi_m)$, where χ_m is the magnetic susceptibility.

$$c=\frac{1}{\sqrt{\varepsilon_o\mu_o}} \qquad v=\frac{1}{\sqrt{\varepsilon\mu_o}} \qquad n=\frac{c}{v}=\sqrt{\frac{\varepsilon}{\varepsilon_o}}$$

We should review the dielectric and associated polarization vector from intro physics. In some intro courses they also talk about the polarization vector. I will derive everything for you. It will not take long.

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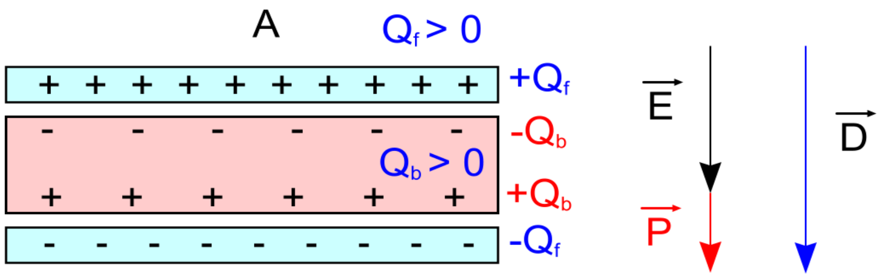
We place a material that does not conduct electricity between two plates that have charge. Such a material is called a **dielectric**. The plates on the other hand are conducting and have free charges, i.e., the charges can roam around. The charges in the dielectric are bound, but noticed they are nudged a little producing an opposing field in the dielectric. Three vector fields appear: **E**, **P**, and **D**. The field **D** is due to the free charges, **E** is due to the total, and **P** is produced by the bound charges AND defined opposite to the usual electric field convention. The P field points from the negative charges to the positive!



Start with

$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q_{free}}{\varepsilon}.$$

Then

$$\varepsilon \oiint \vec{E}\cdot d\vec{A} = Q_{free}$$

Define **D** next

$$\vec{D} \equiv \varepsilon \vec{E} \quad \text{so that now} \quad \oiint \vec{D}\cdot d\vec{A} = Q_{free} \equiv Q_f$$

Note our former definition

$$\varepsilon = \varepsilon_o(1+\chi_e) \quad \text{gives} \quad \vec{D} \equiv \varepsilon \vec{E} = \varepsilon_o \vec{E} + \varepsilon_o \chi_e \vec{E}.$$

The last piece is due to the bound charges.

Note again that the polarization vector **P** points in the same direction as **E** and **D**.

$$\vec{P} \equiv \varepsilon_o \chi_e \vec{E} \quad \text{so that} \quad \vec{D} = \varepsilon_o \vec{E} + \vec{P}.$$

Then

$$\oiint \vec{D}\cdot d\vec{A} = Q_f \quad \Rightarrow \quad \oiint \vec{D}\cdot d\vec{A} = \varepsilon_o \oiint \vec{E}\cdot d\vec{A} + \oiint \vec{P}\cdot d\vec{A}$$

Bur **P** produces bound charge and the free charge producing the **D** field is

$$Q_f = Q_{total} - Q_b.$$

Therefore we arrive at the following.

$$\oiint \vec{D}\cdot d\vec{A} = Q_{free} \qquad \varepsilon_o \oiint \vec{E}\cdot d\vec{A} = Q_{total} \qquad \oiint \vec{P}\cdot d\vec{A} = -Q_b$$

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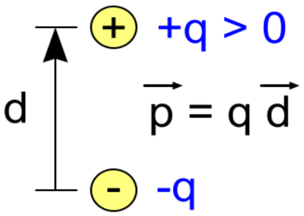
Summary equation: $\vec{D} = \varepsilon_o \vec{E} + \vec{P}$

Let's examine the nature of **P**.

Dimensions for **P** are $[P] = [\varepsilon_o][E]$.

From $\varepsilon_o \oiint \vec{E} \cdot \vec{dA} = Q_{total}$, $[\varepsilon_o][E] = \dfrac{[Q]}{[A]} = \dfrac{C}{m^2}$.



But the simplest **dipole** we learn in intro physics is defined as

$$\vec{p} = q\vec{d} \, ,$$

i.e., you multiply the charge by a vector pointing from minus to plus. However, this dipole definition has units

$$[p] = [q][d] = C \cdot m \, .$$

Perfect! Since there are many dipoles in the dielectric, we would like to multiply by the number of dipole molecules with charge q per unit volume n_q . Then big **P** is then

$$\vec{P} = n_q \vec{p} \qquad \text{and} \qquad [P] = \left[n_q\right][p] = \frac{1}{m^3} C \cdot m = \frac{C}{m^2} \, .$$

P2. Index of Refraction and P. We have the following equations from the last section.

$$\vec{P} \equiv \varepsilon_o \chi_e \vec{E} \qquad \varepsilon = \varepsilon_o (1 + \chi_e) \qquad n = \frac{c}{v} = \sqrt{\frac{\varepsilon}{\varepsilon_o}} \qquad \vec{P} = n_q \vec{p} \qquad \vec{p} = q\vec{d}$$

From the first equation, $\varepsilon_o \chi_e = \dfrac{P}{E}$. Then, $\varepsilon = \varepsilon_o (1 + \chi_e) = \varepsilon_o + \dfrac{P}{E} = \varepsilon_o + \dfrac{n_q p}{E}$.

Then $n^2 = \dfrac{\varepsilon}{\varepsilon_o} = \dfrac{1}{\varepsilon_o}(\varepsilon_o + \dfrac{n_q p}{E})$ with $n^2 = 1 + \dfrac{n_q p}{\varepsilon_o E}$. Fascinating detail!

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P3. Oscillating Bound Charges. Before we delve into the Lorentz oscillator model I would like to show you a powerful way to make an estimate of the natural frequency of vibration for bound charges. The ability to make estimates is a very important part of any subject!

a) Oscillating Electrons. First consider an outer electron in an atom. I am going to make my estimate from two things I remember from chemistry, a clever use of units I learned by being a particle physicist, some intro physics, and some dimensional analysis as my guide. These ingredients are listed below in order to outline the strategy.

- The energy needed to rip off an electron from a hydrogen atom in electron volts.
- An estimate of atomic dimensions for the amplitude oscillations using the Bohr atom.
- The potential energy of a spring system.
- Slick units for mass to help me handle electron volts.
- Some dimensional analysis.

I did not know I needed the above until I started to work out the estimate. So you can appreciate the exploring and use of critical thinking, I will proceed from scratch. I know that the energy of an oscillation system that obeys Hooke's Law has the following equations

$$F = -kx \quad \text{(no friction)}$$

$$x(t) = A\cos(\omega_o t) + B\sin(\omega_o t)$$

$$\omega_o = \sqrt{\frac{k}{m}} \qquad \omega_o = 2\pi f_o \qquad k = m\omega_o^2$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = 0 + \frac{1}{2}kA^2 ,$$

where A is the maximum amplitude of the oscillation. At the maximum extremes, the velocity is zero as the mass turns to go back.

I can get the frequency ω_o if I know E, k, and A since

$$E = \frac{1}{2}kA^2 \quad \Rightarrow \quad E = \frac{1}{2}m\omega_o^2 A^2 \quad \Rightarrow \quad \omega_o^2 = \frac{2E}{mA^2} \quad \Rightarrow \quad \omega_o = \sqrt{\frac{2E}{mA^2}}$$

Well m is easy, the mass of the electron. What about E and A?

I learned in physics/chemistry that it takes 13.6 eV to rip off an electron from the hydrogen atom. I was also advised to know this fact for the physics GRE. Also, from the Bohr model of hydrogen I recall that the electron in its lowest energy state in the semiclassical Bohn model is 0.5 Angstroms, also good to know for the GRE. So to get a rough estimate for the order of magnitude for the oscillation frequency, we take

$$E = 1\ \mathrm{eV}\ .$$

By the way, an electron volt is the energy that an electron gains by accelerating through an electric field that has a potential of 1 volt.

For the amplitude we use 1 ångstroms (1 Å) to place the order of magnitude for distances..

$$E = 1\ \ \mathrm{eV} \qquad A = 1\,\text{Å} = 10^{-10}\,\mathrm{m} \qquad m = ?$$

I don’t remember the mass of the electron in kilograms. But I remember it with the units they use in my profession of particle physics. They take the energy equivalent from Einstein’s

$E = mc^2 = 0.511\ \mathrm{MeV}$. Then $m = \dfrac{0.511\ \mathrm{MeV}}{c^2}$. So now we are ready.

$$E = 1.4\ \mathrm{eV} \qquad A = 0.05 \cdot 10^{-10}\,\mathrm{m} \qquad m = \frac{0.511\ \mathrm{MeV}}{c^2}$$

$$\omega_o = \sqrt{\frac{2E}{mA^2}} \qquad \Rightarrow \qquad \boxed{\omega_o = \frac{1}{A}\sqrt{\frac{2E}{m}}}$$

$$\omega_o \approx \frac{1}{10^{-10}\,\mathrm{m}}\sqrt{\frac{2(1\ \mathrm{eV})}{(0.5\ \mathrm{MeV/c^2})}} = \frac{1}{10^{-10}\,\mathrm{m}}\sqrt{\frac{2\ \mathrm{eV)c^2}}{0.5\ \cdot 10^6\ \mathrm{eV}}}$$

$$\omega_o \approx \frac{10^{10}}{\mathrm{m}}\sqrt{\frac{4(3\cdot 10^8\ \mathrm{m/s})^2}{10^6}} = 10^{10}\sqrt{\frac{4\cdot 9(10^8)^2}{10^6}}\ \frac{\mathrm{radians}}{\mathrm{s}}$$

$$\omega_o \approx 10^{10}\cdot 2\cdot 3\cdot\sqrt{\frac{10^{16}}{10^6}} = 6\cdot 10^{10}\sqrt{10^{10}}$$

$$\omega_o \approx 6\cdot 10^{15}$$

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$$\omega_o = 2\pi f_o \approx 6\cdot 10^{15} \quad => \quad \omega_o \approx 6 f_o \approx 6\cdot 10^{15}$$

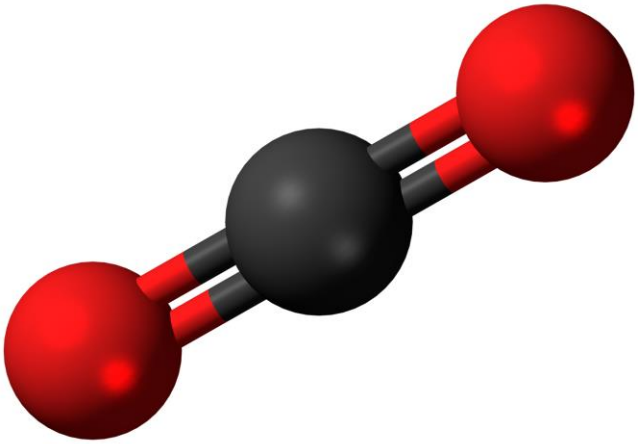
$$f_o \approx 10^{15}\ \text{Hz}$$

$$\lambda_o = \frac{c}{f_o} = \frac{3\cdot 10^8}{10^{15}} = 3\cdot 10^{-7}\,\text{m} = 300\cdot 10^{-9}\,\text{m}$$

$$\boxed{\lambda_o = 300\ \text{nm}}$$

We are in the ultraviolet (UV) for Oscillating Bound Electrons.

b) Oscillating Atoms. What about atoms? Below is carbon dioxide, CO_2.



Carbon Dioxide Model
Carbon is Black, Oxygen is Red
Wikipedia: Jynto.
Released into the Public Domain

The entire oxygen atom can vibrate toward and away from the central carbon atm.

Since the forces are electrical in nature for both electron oscillations and atomic oscillations, the frequency can be estimated from the relation mass of the larger atom to the mass of the electron. Note that

$$\omega_o = \frac{1}{A}\sqrt{\frac{2E}{m}} \qquad \text{means that} \qquad \omega_o \sim \sqrt{\frac{1}{m}} \ .$$

Therefore I take $$\frac{\omega_o^{atom}}{\omega_o^{electron}} = \frac{\sqrt{\dfrac{1}{m_{atom}}}}{\sqrt{\dfrac{1}{m_{electron}}}} = \sqrt{\frac{m_{electron}}{m_{atom}}} = \sqrt{\frac{m_e}{m_{atom}}} \ .$$

As expected, the frequency of the atomic oscillations are lower due to the larger inertia, i.e.,

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$$m_{atom} >> m_e \quad \Rightarrow \quad \omega_o^{atom} << \omega_o^{electron} .$$

We can find $\omega_o^{atom} = \omega_o^{electron}\sqrt{\dfrac{m_e}{m_{atom}}}$ from the ratio $\dfrac{m_e}{m_{atom}}$.

When making quick estimates, part of the fun is not looking things up.

I recall a chemistry teacher telling us that 1836 electron masses equals the mass of the proton and that number stuck in my mind a date: 1836. Then later as a particle physicist the following became ingrained in me.

Particle	**Energy Equivalent of Mass**
Electron	0.511 MeV
Proton	938 MeV
Neutron	940 MeV

Note that

$$\frac{m_p}{m_e} = \frac{938}{0.511} = 1836 .$$

The mass of our oscillating oxygen atom ${}^{16}_{8}O$ is virtually due to the 8 protons and 8 neutrons.

$$m_{oxygen} = 8m_p + 8m_n + 8m_e \approx 16m_p$$

$$\text{Symbol} = {}^{A}_{Z}X$$

Number Protons + Neutrons
Number Protons

A = mass number (total number of protons and neutrons), also Nucleon Number
Z = atomic number (total number of protons)

$$\omega_o^{oxygen} = \omega_o^e\sqrt{\frac{m_e}{m_O}} = \omega_o^e\sqrt{\frac{m_e}{16m_p}} = \omega_o^e\sqrt{\frac{m_e}{16\cdot 1836m_e}}$$

$$\omega_o^O = \omega_o^e\sqrt{\frac{1}{16\cdot 1800}} = \omega_o^e\sqrt{\frac{1}{16\cdot 9\cdot 2\cdot 100}} = \omega_o^e\frac{1}{4\cdot 3\cdot\sqrt{2}\cdot 10}$$

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$$\omega_o^O = \frac{1}{\sqrt{2}\cdot 120}\omega_o^e = \frac{1}{1.4\cdot 120}\omega_o^e$$

$$\omega_o^O = \frac{1}{1\cdot 120 + 0.4\cdot 120}\omega_o^e$$

$$\omega_o^O = \frac{1}{120+48}\omega_o^e$$

$$\omega_o^O = \frac{1}{168}\omega_o^e$$

$$\omega_o^O = \frac{1}{170}\omega_o^e$$

$$\lambda_o^O = 170\cdot\lambda_o^e = 170\cdot 300\text{ nm} = 5100\text{ nm}$$

$$\boxed{\lambda_o^O = 5000\text{ nm}}$$

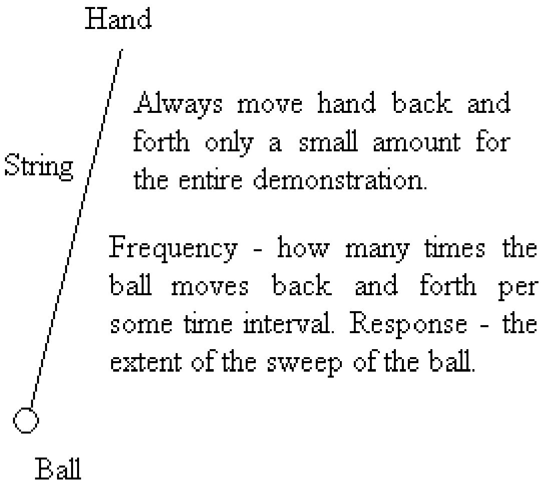
We are in the infrared (IR) for Oscillating Bound Atoms.

P4. Driven Oscillations Resonance. In the last section we considered natural oscillations of bound charged particles. What about forcing these charges to oscillate in step with an incoming EM wave? Such oscillations are called **driven oscillations**. The incoming EM waves with frequency ω drive bound charges forcing them to oscillate at the frequency of the EM wave (ω), But should the EM wave frequency be at the natural frequency ω_o of vibration of the bound charge, the bound charge goes wild. This phenomenon is called resonance.

A most beautiful way to understand resonance is by resonance in mechanics. A very simple example is the pendulum: ball on the end of a string. And you can do this demonstration to any non-science friends or family.

Tape a small ball to the end of a string. Grab the end of the string opposite the ball. Shake your hand back and forth, keeping your sweep within a short space (1 or 2 cm). You may have a friend bracket this space with a thumb and forefinger for you. Can you find the frequency that makes the ball respond most dramatically? Your driving frequency is then called the *resonance frequency*. You are driving the pendulum at the frequency it likes to swing.

Stop, pull back the ball, and let it go so that it swings on its own. Isn't this the same frequency you used earlier? It will be unless you do the experiment in a thick medium like "oil."



Always move hand back and forth only a small amount for the entire demonstration.

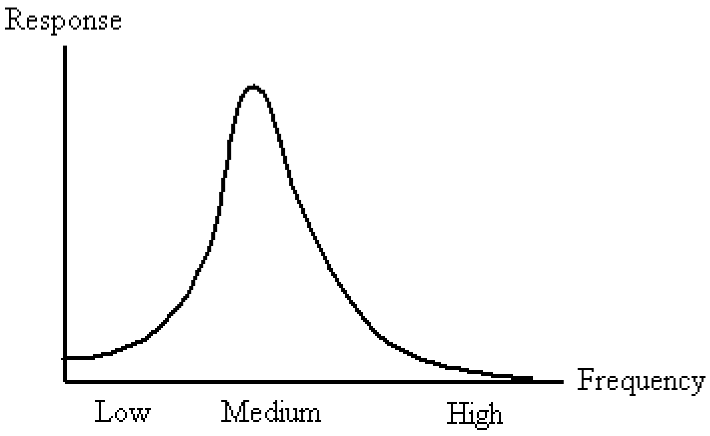
Frequency - how many times the ball moves back and forth per some time interval. Response - the extent of the sweep of the ball.

Hand Driving a Pendulum.

Get a feel for the ball swinging on its own by counting or having someone clap each swing. Now stop the ball and start driving it from rest at this frequency. The ball will gradually increase its swing until it reaches a maximum as you continue to drive the pendulum at the resonance frequency.

Resonance occurs when the ball responds with maximum swing. Resonant vibrations are also called ***sympathetic vibrations*** (the system "is in sympathy" with your vibrations). Compare the responses of the ball (total extent of swing) for different driving frequencies (low, high, medium). Note that the ball moves a little with a low-frequency driving force, and hardly moves at all when driven at high frequency. Somewhere at a medium frequency, the response is greatest (resonance).

Now we are going to make a graphical sketch of our results. We plot a graph of response (vertical axis) versus frequency (horizontal axis). A low response is found at low frequencies. When you shake the pendulum slowly, the ball moves very little. When you shake the string rapidly, the response is even less. You change directions so rapidly that the ball cannot respond quickly enough. So it just sits there. However, at a medium or intermediate frequency, the ball responds the greatest. The frequency that gets you the peak response is called the **resonance frequency**.



We expect resonances at the natural frequencies of vibration for electrons and atoms.

In the ultraviolet (UV) for Oscillating Bound Electrons.
In the infrared (IR) for Oscillating Bound Atoms.

Let’s delve more into the UV and IR regions of the EM spectrum.

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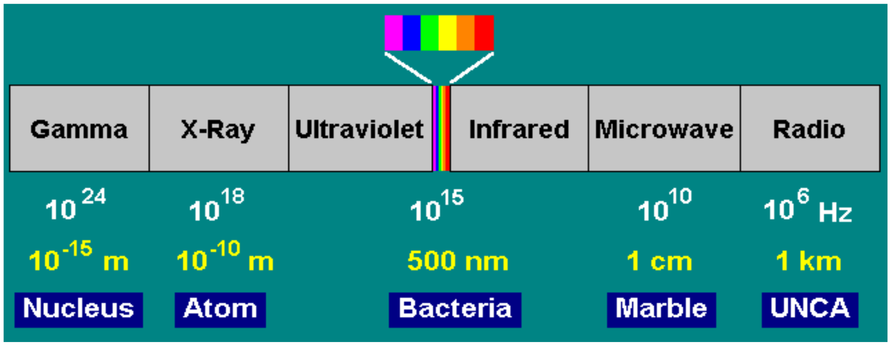
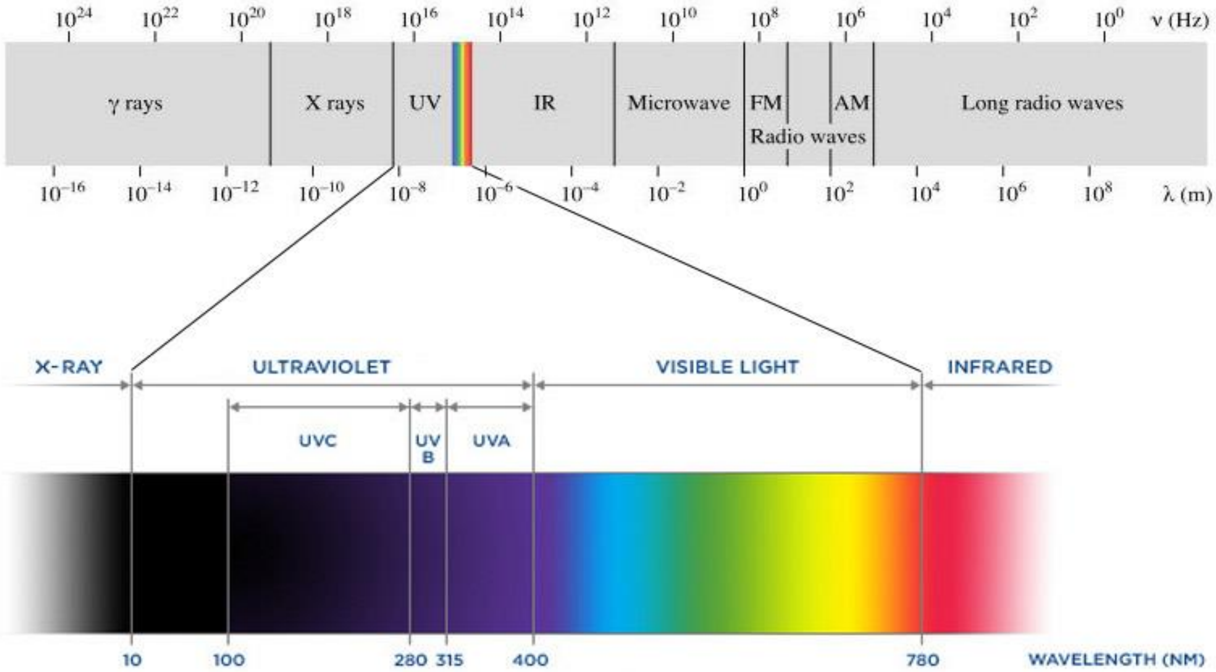
The UV





Courtesy David Darling

Ultraviolet Region	Wavelength	Atmosphere	Effects
EUV	10 - 100 nm	Extreme UV Absorbed by Ionizing the Ionosphere	Very Harmful to Skin and Eye
UVC	100 - 280 nm	All Absorbed with 200 – 280 nm by the Ozone Layer	Very Harmful to Skin and Eye
UVB	280 - 315 nm	Mostly Absorbed by Ozone (Some Reaches Earth)	Causes Suntan and Sunburn (Skin/Eye Protection Advised)
UVA	315 - 400 nm	Reaches Earth	Can Suntan, No Sunburn (Eye Protection is Good Idea)

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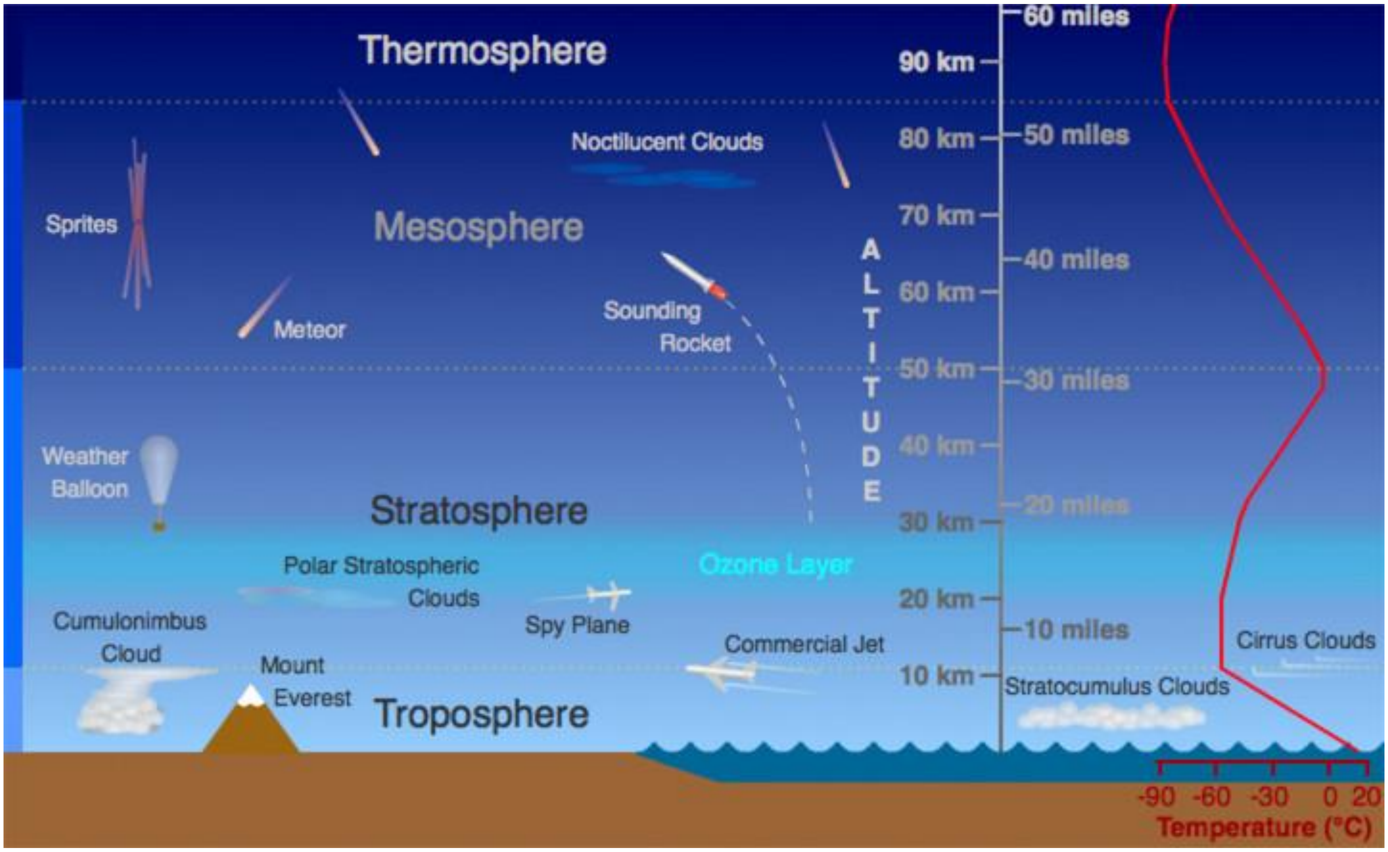
Ultraviolet light (UV) can also be classified as in the table below.

Near UV	NUV	300 – 400 nm
Middle UV	MUV	200 – 300 nm
Far UV	FUV	100 – 200 nm
Extreme UV	EUV	10 – 100 nm

The ozone layer is shown below.



Layers of the atmosphere: troposphere, stratosphere, mesosphere and thermosphere. [Courtesy UCAR](). Credit: Randy Russell, UCAR.

More Atmospheric Sciences!

From this rich physics and chemistry we have great examples in the interdisciplinary area of Atmospheric Science and Environmental Science.

The Ozone layer and Ozone Holes.

Carbon Dioxide and the Greenhouse Effect.

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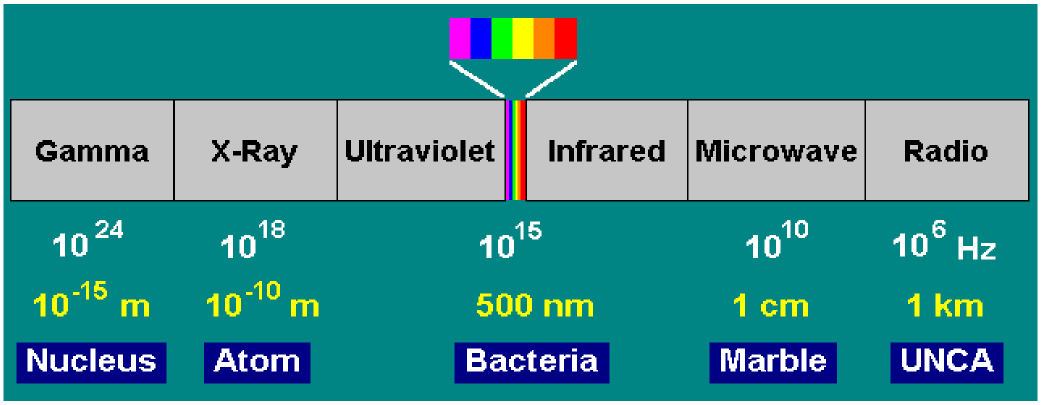
Time to add Environmental Studies to our List of Interdisciplinary Topics.

- ENVR, Environmental Studies (Science) – Ozone, Greenhouse Effect

And we add today to ATMS: Why is the sky blue? And ozone.

1. ATMS, Atmospheric Science (Meteorology) – mirages, rainbows, **ozone, blue sky**
2. ASTR, Astronomy – telescopes, solar spectrum, eclipses
3. BIOL, Biology – the optics of the human eye (cornea and eye lens)
4. CHEM, Chemistry – light & electron transitions (Hydrogen, Sodium, Mercury, Cadmium)
5. CSCI, Computer Science – creating a computer visualization for the sine wave
6. **ENVR, Environmental Science – ozone, greenhouse effect, global warming**
7. GEOL, Geology – diamond, birefringence in gems
8. MAG, Magic – illusions based on optics
9. MATH, Mathematics – algebra, geometry, trigonometry, and calculus
10. ENGR, Engineering – design of mirrors (wide-angle, vanity) and lenses
11. MED, Medicine – visual acuity, prescribing eyeglasses for myopia and hyperopia
12. PHOT, Photography – f/#, aperture, camera lenses
13. PHYS, Physics – Laws of Reflection, Refraction, Spherical Mirrors, and Lenses

The IR



The Spectrum of Infrared Radiation

EM Radiation	Sub-Region (IR)	Wavelength (nm)	Atmosphere
Infrared (IR)	Near IR	700 - 3000	
	Mid IR	3000 - 6000	Most is Absorbed by Atmosphere
	Far IR	6000 - 15,000	
	Extreme IR	15,000 - 1,000,000	

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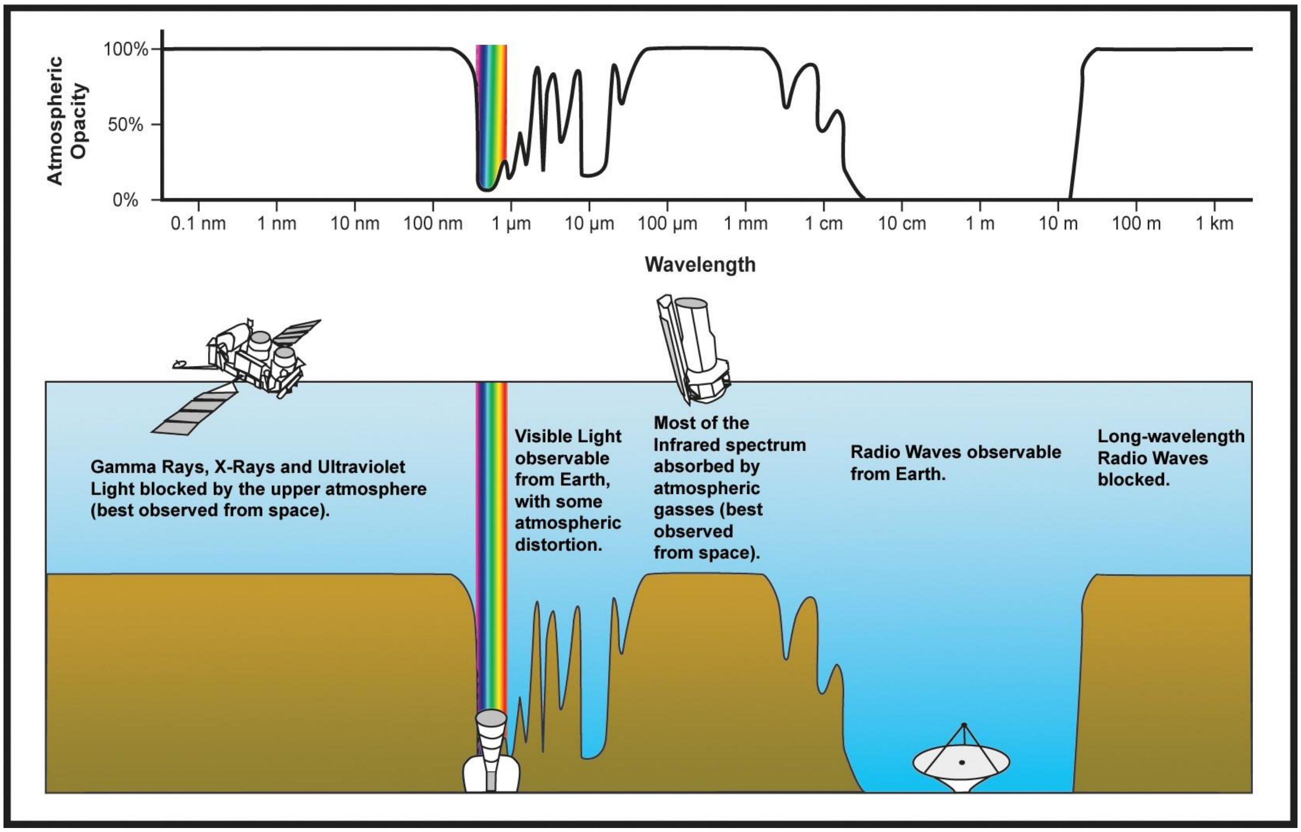
Visible light can easily pass through carbon dioxide gas while infrared has trouble. The frequencies of the infrared light match resonance frequencies of atoms in the carbon dioxide molecule.

The carbon dioxide molecules start vibrating at these infrared frequencies, re-emitting much of the IR in all directions. Thus the IR is prevented from making a free passage through the carbon dioxide either coming from the Sun or trying to leave from the Earth. The resonance frequencies for carbon dioxide have corresponding wavelengths in the ball park of 5000 nm, like we found in the previous section.

Absorption of the EM Spectrum by the Atmosphere



Courtesy NASA

Two Windows of the Atmosphere: visible spectrum and some radio wave.

The narrow absorption peak near 5 μm = 5000 nm is due to CO_2.

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UNCA's Greenhouse

Photo by Prof. Ruiz, Fall 1998

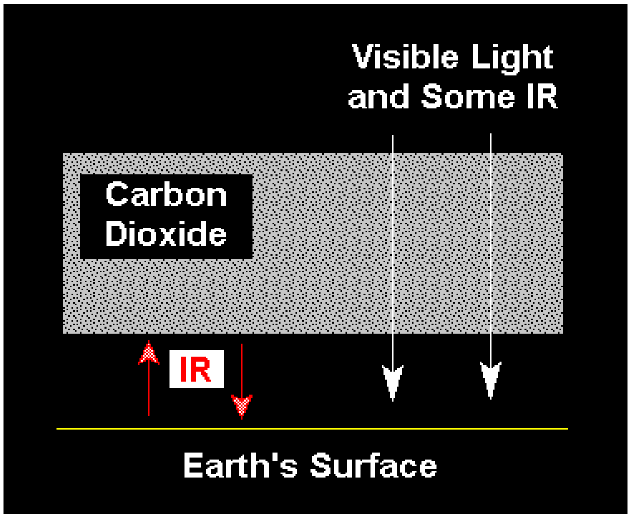
A ***greenhouse*** is an enclosure of mostly glass to allow light to reach plants and keep them warm in the winter. The greenhouse functions by preventing rising hot air from escaping. The enclosure also prevents cold outside air from mixing with the air inside.

However, the ***greenhouse effect*** is different! The common use of the term ***greenhouse effect*** refers to the phenomenon where

1) visible light and some infrared radiation pass through the Earth's atmosphere and

2) the carbon dioxide in the atmosphere traps some infrared trying to leave from the Earth's surface.

Visible light can easily pass through carbon dioxide gas while infrared has trouble. The frequencies of the infrared light match resonance frequencies of atoms in the carbon dioxide molecule.



The diagram illustrates the greenhouse effect described on a previous page. Carbon dioxide is referred to as a greenhouse gas due to this effect. Water vapor is also a greenhouse gas.

In an actual greenhouse the separation of cold and warm air is the dominant factor in protecting the plants. So the "greenhouse effect" was named incorrectly. If you want an excellent example of the "greenhouse effect" with glass, look at those cars in the ***UNCA*** parking lot in the photo at the top of this page. During the summer, visible light easily passes through the glass windows, along with some IR. The interior of the car heats up. The IR has difficulty going through glass. Therefore, much heat stays trapped in the car, building up. On hot summer days, this can occur rapidly. Never leave a pet animal in the car during the summer, even with the window cracked. Here is your professor's encounter with the "greenhouse effect" in a car.

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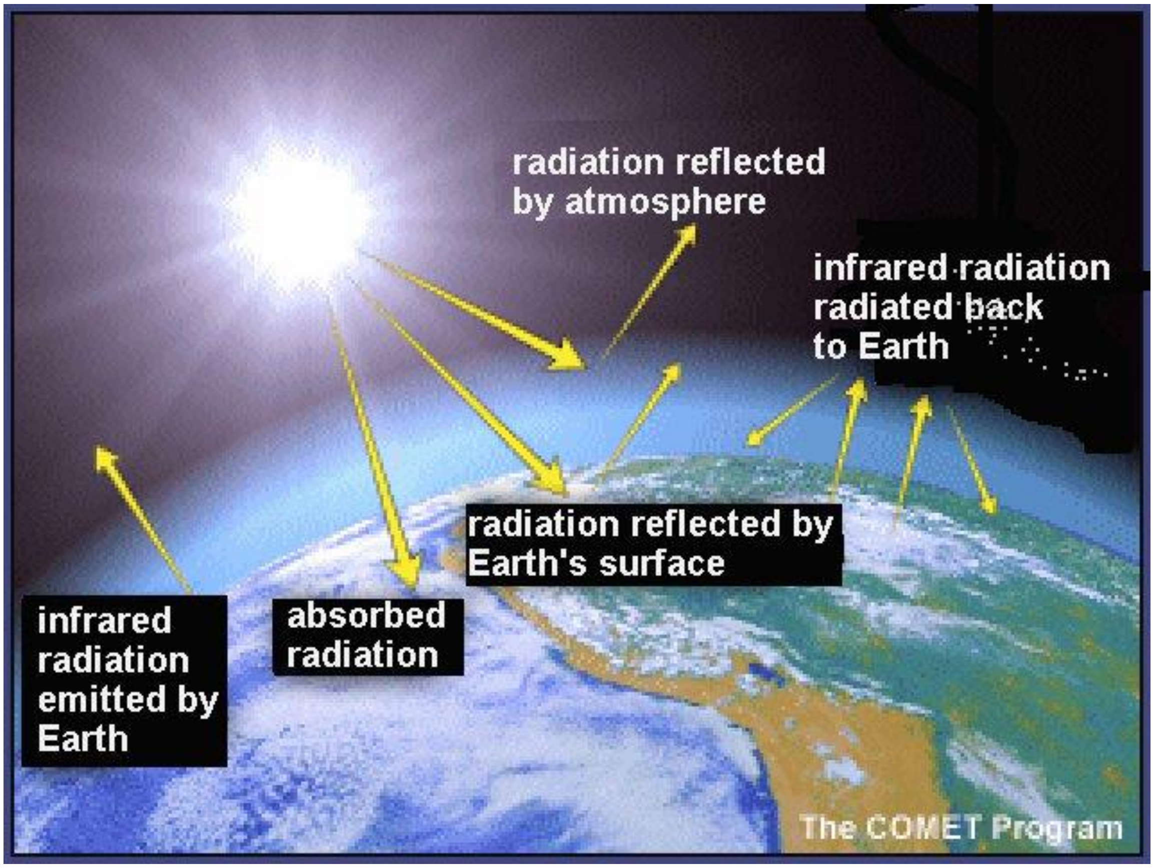
"As a graduate student many years ago I had the good fortune of taking care of a house owned by the Vice President of Martin Marietta, Charles Leithauser, in the DC area. One day I was driving his wife's car on a hot summer day. The beltway traffic was jammed. Then the car stalled and the air conditioner went off. The hot 90-degree day was pumping energy so rapidly into the car that after 10 seconds with the engine off, the heat was getting unbearable.

"I went to roll down the window as I was accustomed to in all the previous inexpensive cars I had driven, looking for the crank. No luck, I couldn't find one. This fancy car had some modern electrical system, and since the engine was off, I couldn't get the window to open. As the time approached the 20-second mark, I started to realize that if I didn't get this car started or the window open real soon, I would be in deep trouble - maybe fatal! With a stroke of luck the car started. I was okay. But the "greenhouse effect" scared the dickens out of me." ***Prof. Ruiz, January 2001, recounting an experience from circa 1975***

Here is a nice figure that summarizes what we have discussed. Note that some IR gets reflected by the atmosphere and does not reach Earth. Also note that some IR passes through the atmosphere coming and going. Can you find where in the diagram the "Greenhouse Effect" is illustrated?

Carbon Dioxide Preventing Solar IR from Reaching the Earth
Carbon Dioxide Trapping IR Radiated from Earth => Global Warming



Courtesy University Corporation for Atmospheric Research (UCAR), Boulder, CO

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As the industrial revolution reached new heights during 1950s, trouble for the ozone layer was beginning. By the early 1950s, everyone was excited about the new ***aerosol spray can***. An aerosol is an airborne collection of small particles in solid or liquid form. New chemicals were used in aerosol cans to provide the pressure to spray the aerosol contents of virtually anything for the consumer. These chemicals are known as ***CFCs*** or ***chlorofluorocarbons***. The CFC is made from **Ch**lorine, **F**luorine, and **C**arbon atoms - thus its name. This chemical was very desirable because it did not react with other stuff

A CFC could be mixed with anything and pressurized. Then, one could propel the mixture out into the environment whereby the desired ingredient would spray where you aimed it. The non-reactive CFCs meanwhile drifted away as an invisible gas.

Thumbnail version (for educational fair use) of image in Book of Knowledge (Grolier 1952)

Below is a quote from 1952 illustrating the hype about this new wonder.

"You press the button at the top of the can and from the nozzle squirts a mist, a spray or a foam. These are three different forms of the new preparations known as 'aerosols,' which can do many a chore around the home. The mist that comes from the nozzle may be perfume or it may be poison that will kill insects. When the aerosol is in the form of a spray, it may be paint to be sprayed on your furniture or a sun-tan lotion for your skin. Whipped cream comes from the nozzle in the form of foam all ready to cover the top of a cake or to add to the pleasure of eating strawberries fresh from the garden. Aerosols can now be found on the shelves of groceries and drugstores." From the annual ***Book of Knowledge Encyclopedia***, 1952 (Grolier published these annual books from 1890 to 1960. Kids grew up reading these fascinating general books of the times.)

But guess what? No one knew then that this very non-reactive compound worked its way up into the stratosphere with bad news. There, CFCs began reacting with the ozone layer - destroying ozone molecules!

The warning came in 1974, twenty years after the surge of aerosols that began in the early 1950s. CFCs do not react with anything - at least, that's what everyone thought in the 1950s. So CFCs were considered truly inert and everyone felt this was great. Well, CFCs do not react until they work their way up to the ozone layer. Then they start eating up ozone. The chemist Sherwood Rowland (1927-2012) made this discovery in 1974.

It was discovered by Sherwood Rowland and his colleague in the 1970s that the ozone layer gets broken down from ***CFCs***. These are molecules of chlorine, fluorine, and carbon atoms which we refer to as chlorofluorocarbons or CFCs for short. The former commercial use of CFCs in aerosol spray cans has been stopped but CFCs are still used in air conditioning and manufacturing.

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Originally Rowland had no idea that his work would have an environmental impact and that this research would earn him a shared Nobel Prize in Chemistry in 1995. He shared the prize with Mario J. Molina and Paul J. Crutzen “for their work in atmospheric chemistry, particularly concerning the formation and decomposition of ozone” (the Nobel Prize Committee). Scientists for the most part pursue research for the fun of the research, i.e., finding the answer to some puzzle of nature. Unexpected connections can often be found with practical applications in society.

The Famous Chemist Visits UNCA, Fall 2004. Who is the guy at the left?

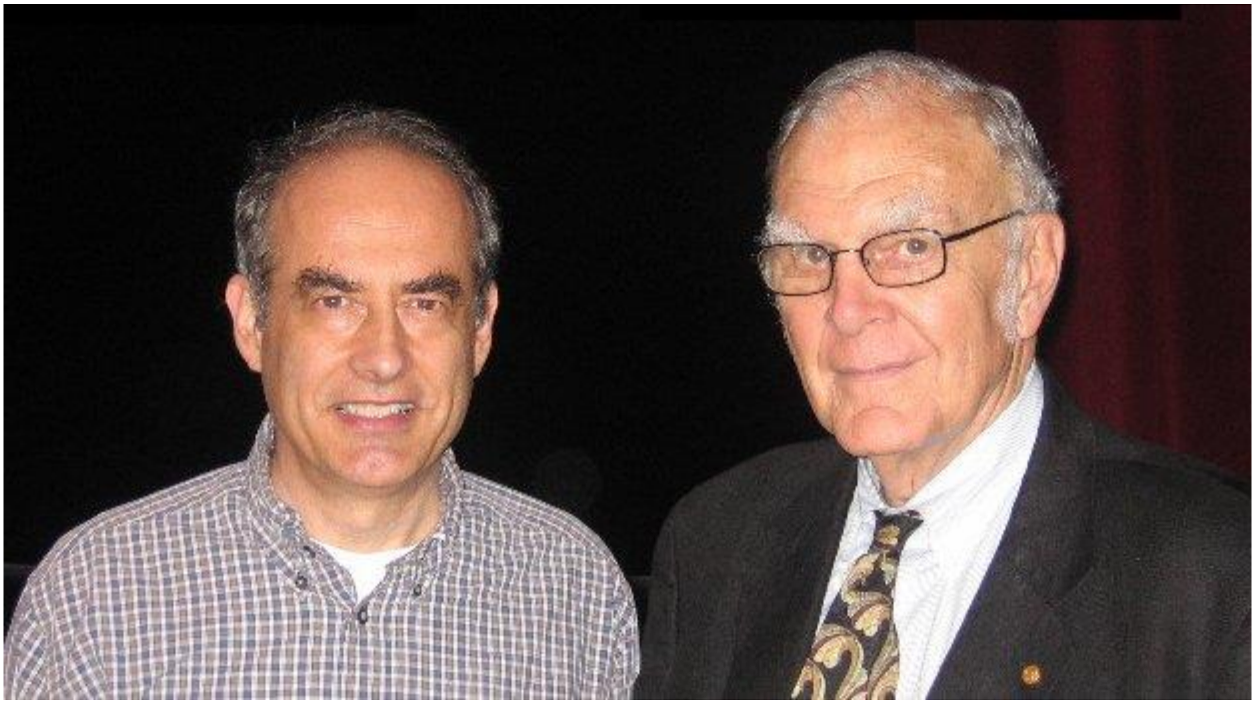
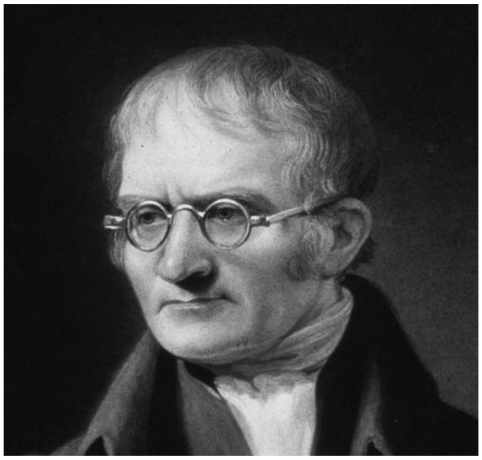


Photo by Chemistry Prof. John Stevens, UNCA Lipinsky Auditorium, September 9, 2004

In his talk at UNCA, Prof. Rowland mentioned a line of his teachers goes back to the chemist Dalton.



John Dalton (1766 – 1844)
Chemist

Atomic Theory of Matter (1803)

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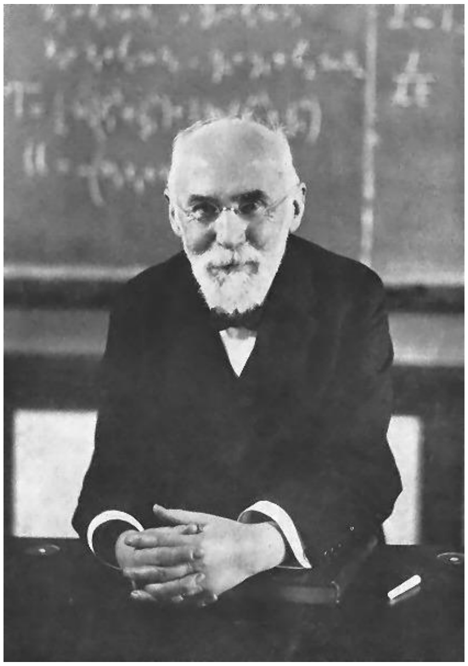
P5. The Lorentz Oscillator Model.



Hendrik Antoon Lorentz (1853 – 1928)
Public Domain Photo.
Main Contributions Below.

1. Nobel Prize 1902 with Pieter Zeeman for the experimental discovery and theoretical analysis of the Zeeman effect.

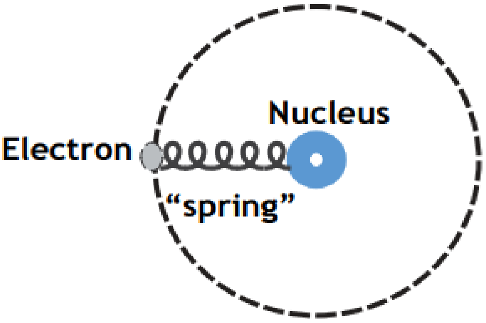
2. Known for his Lorentz Transformation proposed in the pre-relativity era, a coordinate transformation that cane be elegantly derived from Einstein's Theory of Special Relativity. The Lorentz transformation between the rest frame K and moving frame K' with velocity v:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} .$$

3. Known for his elegant derivation of the Lorentz force: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ in E&M.

4. Lorentz Oscillator Model. Classical as it predates quantum mechanics.

Remember how we said that all equilibrium problems can be approximated as a harmonic oscillator with small oscillating displacements? Think of an outer electron undergoing such oscillations in response to an incoming EM waves.



Effective Spring for Oscillating Outer Electron Responding to Incoming EM Wave

Courtesy MIT OpenCourseWare
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MIT Source File

We now once again set up the harmonic oscillator problem: Newton's Second Law and Hooke's Law.

$$F = ma \qquad F = -kx$$

Then, using the notation $a = \ddot{x}$ => $m\ddot{x} = -kx$.

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Since we solved this differential equation before using the regular methods with cosines and sines, let's solve it now with our more sophisticated phasor method: $x = Ae^{i\omega_o t}$.

$$m\ddot{x} = -kx \quad \Rightarrow \quad mA(i\omega_o)^2 e^{i\omega_o t} = -kAe^{i\omega_o t} \quad \Rightarrow \quad -mA\omega_o^2 e^{i\omega_o t} = -kAe^{i\omega_o t}$$

$$-m\omega_o^2 = -k \quad \Rightarrow \quad m\omega_o^2 = k \quad \Rightarrow \quad \omega_o^2 = \frac{k}{m} \quad \Rightarrow \quad \omega_o = \sqrt{\frac{k}{m}}$$

These results are all familiar from before.

These equations describe the natural frequency of vibration for the charge.

Now we send in an EM wave $E_o e^{i\omega t}$ and drive the charge.

A charge q responds to an electric field $\vec{E}$ of our namesake Lorentz

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}.$$

Wait. A magnetic field accompanies the electric field.
Shouldn't the moving charge then experience a magnetic force too?

Remember that the accompanying magnetic field obeys

$$\frac{E}{B} = c.$$

The magnetic force is then $\sim qvB$.

Using $B = \dfrac{E}{c}$, then $F_B = qvB = qv\dfrac{E}{c} = qE\dfrac{v}{c}$.

Since we are in a non-relativistic realm, $v << c$, leading to

$$F_B = qE\frac{v}{c} << qE = F_E \text{, i.e., } F_B << F_E.$$

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So we send in the EM wave and only consider $E_o e^{i\omega t}$ driving the charge.

Driving force equation:

$$F = ma \quad \text{and} \quad F = -kx + E_o e^{i\omega t} .$$

$$m\ddot{x} = -kx + qE_o e^{i\omega t}$$

$$m\ddot{x} + kx = qE_o e^{i\omega t}$$

For an electron, $q = -e$, where $e > 0$.

$$m\ddot{x} + kx = -eE_o e^{i\omega t}$$

$$\ddot{x} + \frac{k}{m} x = -\frac{eE_o}{m} e^{i\omega t}$$

Substitute $\omega_o = \sqrt{\frac{k}{m}}$, i.e., $\frac{k}{m} = \omega_o^2$ in $\ddot{x} + \frac{k}{m} x = -\frac{eE_o}{m} e^{i\omega t}$ to obtain

$$\ddot{x} + \omega_o^2 x = -\frac{eE_o}{m} e^{i\omega t} .$$

Since we are forcing the oscillations like driving our pendulum, the solution is of the form

$$x = x(t) = Ae^{i\omega t} .$$

Then $\dot{x} = i\omega Ae^{i\omega t} = i\omega x$

and

$$\ddot{x} = (i\omega)^2 Ae^{i\omega t} = -\omega^2 Ae^{i\omega t} .$$

Substitute $x = Ae^{i\omega t}$ and $\ddot{x} = -\omega^2 Ae^{i\omega t}$ into $\ddot{x} + \omega_o^2 x = -\dfrac{eE_o}{m} e^{i\omega t}$.

$$-\omega^2 Ae^{i\omega t} + \omega_o^2 Ae^{i\omega t} = -\frac{eE_o}{m} e^{i\omega t}$$

$$(-\omega^2 + \omega_o^2) Ae^{i\omega t} = -\frac{eE_o}{m} e^{i\omega t}$$

$$(\omega_o^2 - \omega^2) Ae^{i\omega t} = -\frac{eE_o}{m} e^{i\omega t}$$

$$(\omega_o^2 - \omega^2) A = -\frac{eE_o}{m}$$

$$A = -\frac{eE_o}{m(\omega_o^2 - \omega^2)}$$

$$x = -\frac{eE_o}{m(\omega_o^2 - \omega^2)} e^{i\omega t}$$

We combine this result with the polarization equations from earlier.

$$n^2 = 1 + \frac{n_e p}{\varepsilon_o E_o} \qquad \vec{p} = q\vec{d}$$

The displacement $\vec{d}$ is our displacement $\vec{x}$.

Therefore, the magnitude $p = qd = qx = -ex$.

$$p = -ex = -e\left[-\frac{eE_o}{m(\omega_o^2 - \omega^2)} e^{i\omega t}\right] = \frac{e^2 E_o}{m(\omega_o^2 - \omega^2)} e^{i\omega t}$$

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$$\vec{P} = n_e \vec{p}$$

$$P = \frac{n_e e^2 E_o}{m(\omega_o^2 - \omega^2)} e^{i\omega t}$$

$$E = E_o e^{i\omega t}$$

$$P = \frac{n_e e^2}{m(\omega_o^2 - \omega^2)} E$$

$$\vec{P} \equiv \varepsilon_o \chi_e \vec{E}$$

$$\varepsilon_o \chi_e = \frac{n_e e^2}{m(\omega_o^2 - \omega^2)}$$

$$\varepsilon = \varepsilon_o (1 + \chi_e)$$

$$\varepsilon = \varepsilon_o + \varepsilon_o \chi_e$$

$$\varepsilon = \varepsilon_o + \frac{n_e e^2}{m(\omega_o^2 - \omega^2)}$$

$$n = \sqrt{\frac{\varepsilon}{\varepsilon_o}} \qquad n^2 = \frac{\varepsilon}{\varepsilon_o} \qquad n^2 = \frac{1}{\varepsilon_o}\left[\varepsilon_o + \frac{n_e e^2}{m(\omega_o^2 - \omega^2)}\right]$$

$$\boxed{n^2 = 1 + \frac{n_e e^2}{\varepsilon_o m(\omega_o^2 - \omega^2)}}$$

DISPERSION! Index of refraction depends on frequency of incoming light!

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P6. Dispersion Revisited. Amazing information appears in this dispersion formula.

$$n^2 = 1 + \frac{n_e e^2}{\varepsilon_o m(\omega_o^2 - \omega^2)}$$

Let's examine three cases.

(1) Low frequencies $\omega << \omega_o$

$$\lim_{\omega \to 0} n^2 = 1 + \frac{n_e e^2}{\varepsilon_o m \omega_o^2} = \text{const}$$

(2) Frequencies $\omega \approx \omega_o$, i.e., near ω_o

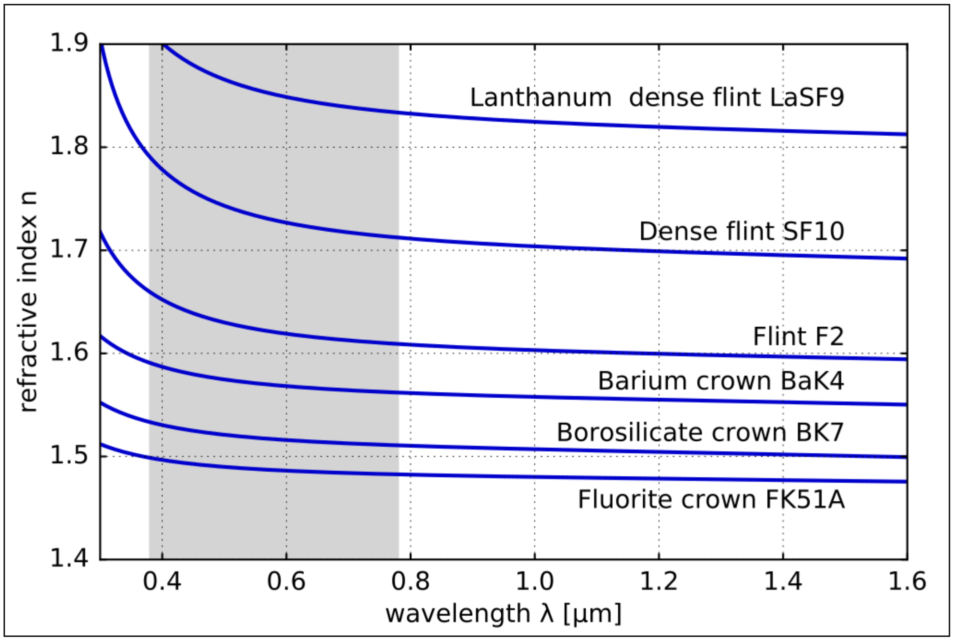
$$\lim_{\omega \to \omega_o} n^2 = \infty \, .$$

We expect this result as we are at resonance: the system goes wild!
But in real situations we have damping, which avoids the infinity.

We now have a theoretical understanding of the flat dispersion curve at large λ.
Remember, low ω means long λ. And we understand the sharp increase as λ nears λ_o.

Remember we estimated electron resonances near 300 nm?

Look at the graph in the UV near 300 nm = 0.3 μm! Compare with the IR.



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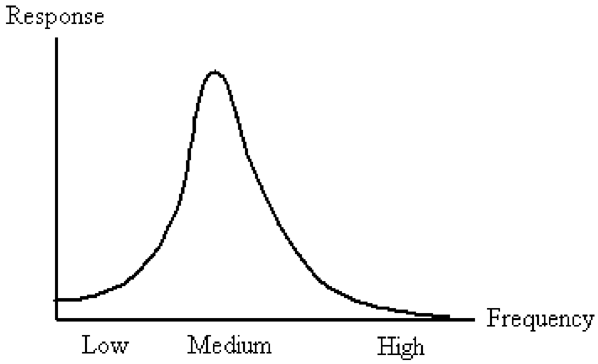
(3) Frequencies $\omega >> \omega_o$.

$$\lim_{\omega\to\infty} n^2 = \lim_{\omega\to\infty}\left[1+\frac{n_e e^2}{\varepsilon_o m(\omega_o^2-\omega^2)}\right] = 1+\lim_{\omega\to\infty}\frac{n_e e^2}{\varepsilon_o m(\omega_o^2-\omega^2)} = 1 .$$

Fascinating.



What happens if you drive the ball on the end of a string at high frequency?

It essentially does nothing. You are driving it too fast.

Response goes to zero. You get nothing. No induced polarization.

Nothing means $n^2 = 1$ and $n = 1$.

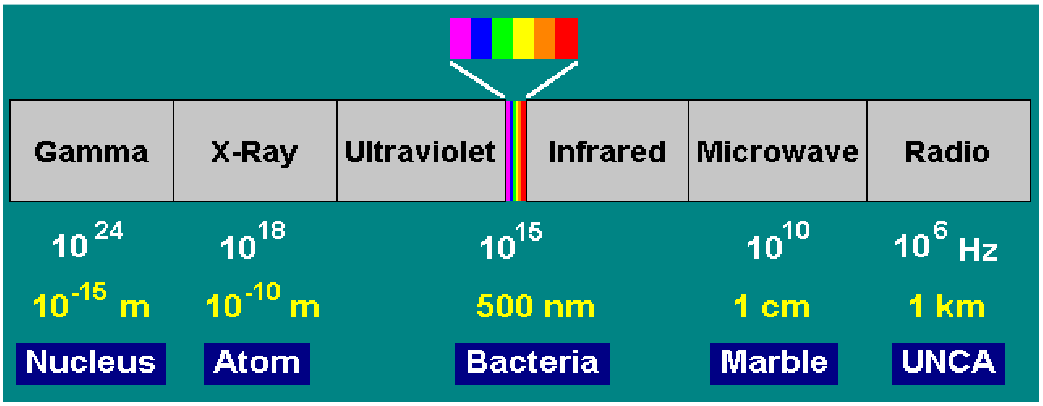
When $\omega >> \omega_o$, we have $\lambda << \lambda_o$.

We are looking for a wavelength such that $\lambda << \lambda_o = 300 \text{ nm}$

What about $\lambda = \dfrac{\lambda_o}{3000} = \dfrac{300}{3000} \text{ nm} = 0.1 \text{ nm} = 1 \text{ Å}$ or even less?

We are in the x-ray and x-rays go straight through glass with no refraction.

Technically these x-rays are called hard x-rays, the ones that go through matter easily.



We covered a great part of the EM spectrum today: IR to X-ray!

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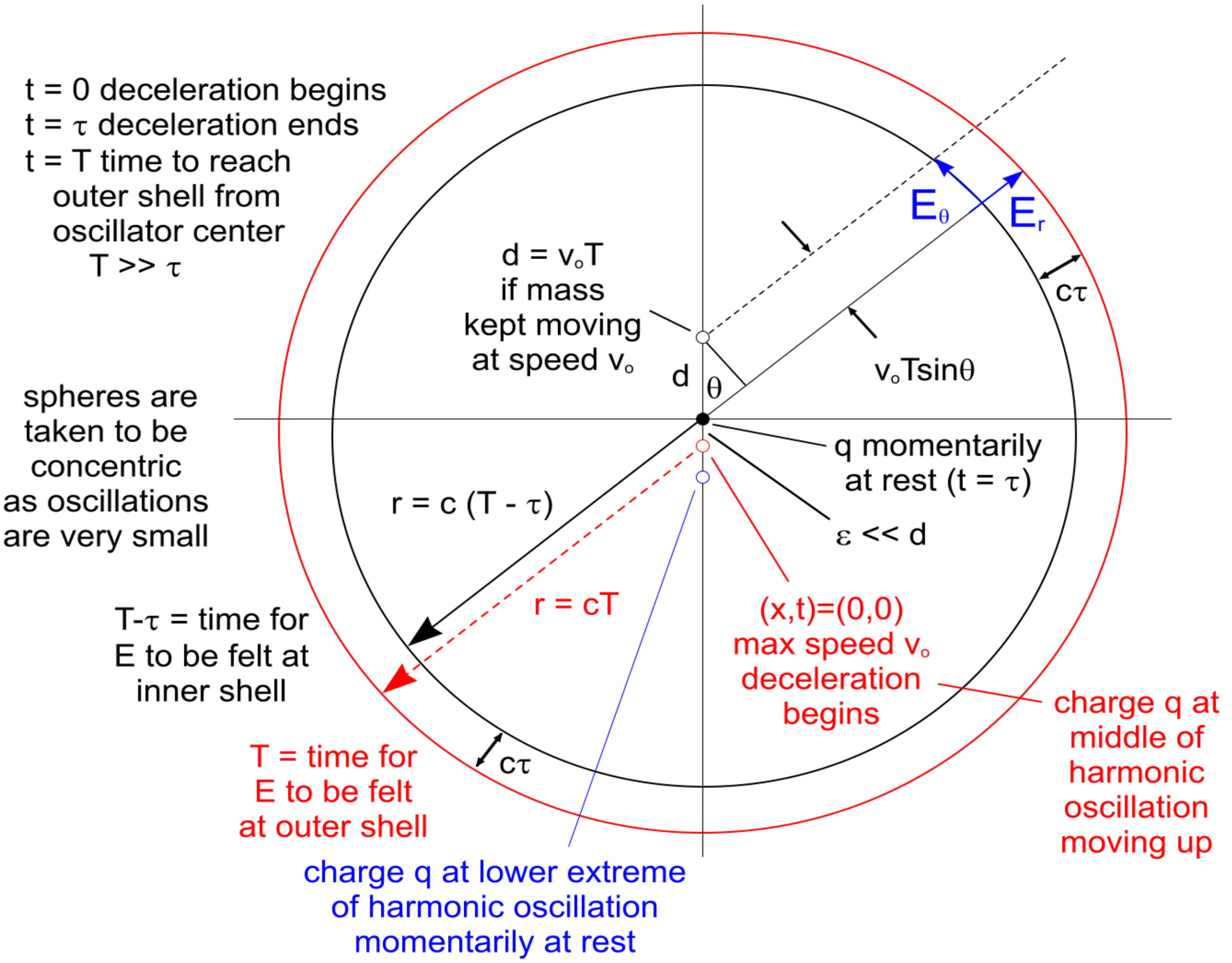
P7. Why is the Sky Blue? In the last section we found

$$x = -\frac{eE_o}{m(\omega_o^2 - \omega^2)} e^{i\omega t}\ .$$

The oscillating bound charge will reradiate EM waves. We need to understand this reradiation in order to complete the analysis.



$$\frac{E_\theta}{E_r} = \frac{v_o T \sin\theta}{c\tau} \qquad E_r = \frac{q}{4\pi\varepsilon_o r^2} \qquad r = cT$$

$$E_\theta = \frac{v_o T \sin\theta}{c\tau} E_r = \frac{v_o T \sin\theta}{c\tau}\frac{q}{4\pi\varepsilon_o r^2} = \frac{v_o}{\tau}\frac{(r/c)\sin\theta}{c}\frac{q}{4\pi\varepsilon_o r^2}$$

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$E_\theta = \frac{v_o}{\tau}\frac{\sin\theta}{c^2 r}\frac{q}{4\pi\varepsilon_o} = \frac{aq\sin\theta}{4\pi\varepsilon_o c^2 r}$ where $a = \frac{v_o}{\tau}$. The acceleration.

With $q = -e$, we have

$$E_\theta = -\frac{ae\sin\theta}{4\pi\varepsilon_o c^2 r} \qquad x = -\frac{eE_o}{m(\omega_o^2 - \omega^2)}e^{i\omega t}$$

$$a = \ddot{x} = (i\omega)^2 x = -\omega^2\left[-\frac{eE_o}{m(\omega_o^2 - \omega^2)}e^{i\omega t}\right] = \frac{\omega^2 eE_o}{m(\omega_o^2 - \omega^2)}e^{i\omega t}$$

$$E_\theta = -\frac{ae\sin\theta}{4\pi\varepsilon_o c^2 r} = -\frac{\omega^2 eE_o}{m(\omega_o^2 - \omega^2)}e^{i\omega t}\frac{e\sin\theta}{4\pi\varepsilon_o c^2 r}$$

$$E_\theta = -\frac{e^2\omega^2 E_o e\sin\theta}{4\pi\varepsilon_o c^2 r m(\omega_o^2 - \omega^2)}e^{i\omega t}$$

$$I \sim E_\theta E_\theta^* \sim \left[\frac{\omega^2}{(\omega_o^2 - \omega^2)}\right]^2 = \frac{\omega^4}{(\omega_o^2 - \omega^2)^2}$$

At low frequencies $\omega << \omega_o$

$$I \sim \omega^4 \sim \frac{1}{\lambda^4}$$

The famous 4th power law!

Shorter wavelengths reradiate (scatter) more, i.e., the blue end of the spectrum.

The Sky is Blue!

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Blue Sky over UNCA (Looking North at Ramsey Library)

Photo by Doc, UNCA Campus, November 2, 2002

I use the following non-math description in my optics course for non-science majors:
PHYS 101 Light and Visual Phenomena.

Why is the Sky Blue? The British physicist Lord Rayleigh (1841-1919) derived the theoretical results that explain why the sky is blue. The key lies in the size of the particles scattering the light and resonance.

Consider a particle smaller than the wavelength of light, i.e., a molecule (or atom). The incoming light causes outer electrons of the molecule to wiggle in step with the light waves. Since the electrons are wiggling and they are charges, they generate new light waves. They do this most "wildly" near the resonance frequency, as you would expect.

The electron resonance frequencies tend to be in the ultraviolet since electrons are lightweight and therefore like to wiggle at high frequencies. The result is that this reradiation favors the wavelengths closer to the ultraviolet. This means much violet and blue getting reradiated all over

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the sky. We call this reradiation ***Rayleigh scattering***. The effect is less and less as wavelengths increase to the red end of the spectrum. Always remember that the blue-end of the spectrum scatters more.

When particles get larger, e.g., through condensation to form mist and clouds, the density causes so many internal reflections that the light coming to your eye contains all the wavelengths. Therefore, mist and clouds appear white. When you observe smoke, you see white when the smoke is thick (like a cloud) and a blue tinge when the smoke is thinned out.

Why is the Sunset Red? When the Sun is low on the horizon the sunlight must travel through much atmosphere. More and more blues are scattered out of the sunlight and now an appreciable amount of greens are gone along with the blues. The remaining light traveling in the direction away from the Sun is therefore rich in yellows and reds. This red light can bounce off clouds to produce a very colorful sunset scene as illustrated below.

Red Colors of Sunset and Crepuscular Rays

'Passion Kindled, Utah' by Randy Wang, Courtesy Randy Wang

The dramatic rays of light in the above photo are called ***crepuscular rays***, rays of sunlight that pass through cloud openings and scatter off mist.

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