M0. The Wave Equation. Light is an electromagnetic wave. Therefore, optics is intimately connected to electricity and magnetism, i.e., electromagnetic theory. We first derive the general wave equation so we can recognize it when we encounter specifically EM waves later. From our last class, we start with a function traveling to the right: $\psi(x,t) = f(x - vt)$. Let $u = x - vt$. Note that $\frac{\partial u}{\partial x} = 1$ and $\frac{\partial u}{\partial t} = -v$. Then we take derivatives in our quest for the "magic" differential wave equation,

$$\frac{\partial \psi(x,t)}{\partial x} = \frac{\partial f(x-vt)}{\partial x} = \frac{\partial f(u)}{\partial x} = \frac{df(u)}{du}\frac{\partial u}{\partial x} = \frac{df(u)}{du} \cdot 1 = \frac{df(u)}{du}$$

$$\frac{\partial \psi(x,t)}{\partial t} = \frac{\partial f(x-vt)}{\partial t} = \frac{\partial f(u)}{\partial t} = \frac{df(u)}{du}\frac{\partial u}{\partial t} = \frac{df(u)}{du} \cdot (-v) .$$

We can now put together the following differential equation from the above. We find

$$\frac{\partial \psi(x,t)}{\partial x} = -\frac{1}{v}\frac{\partial \psi(x,t)}{\partial t} \quad \text{and write} \quad \frac{\partial \psi_R(x,t)}{\partial x} = -\frac{1}{v}\frac{\partial \psi_R(x,t)}{\partial t} ,$$

adding the subscript R for "Right" to emphasize that this wave is traveling down the x axis in the positive direction.

But for the wave traveling to the left, we must have the same equation with the velocity in the negative direction. This reverses the sign in front of v since u in that case would be u = x + vt with f(u) = f(x+vt). Now I know I said to use sin(-kx – vt) in the previous chapter for left-traveling waves per my publication with Perkins, but this sine function is of the form f(x+vt) and we will be okay here.

$$\frac{\partial \psi_L(x,t)}{\partial x} = +\frac{1}{v}\frac{\partial \psi_L(x,t)}{\partial t} .$$

This is not acceptable because now we have two differential equations and there is nothing special about right or left. We want a differential equation where the sign does not matter. So we proceed to the second derivative since we know that second-order differential equations have two independent solutions. We continue from

$$\psi(x,t) = f(x - vt) \quad \text{and} \quad u = x - vt ,$$

$$\frac{\partial\psi(x,t)}{\partial x} = \frac{df(u)}{du} \quad \text{and} \quad \frac{\partial\psi(x,t)}{\partial t} = -v\frac{df(u)}{du},$$

and take the second derivatives with respect to x and t.

$$\frac{\partial^2\psi(x,t)}{\partial x^2} = \frac{\partial}{\partial x}\frac{df(u)}{du} = \frac{d^2 f(u)}{du^2}\frac{\partial u}{\partial x} = \frac{d^2 f(u)}{du^2}$$

$$\frac{\partial^2\psi(x,t)}{\partial t^2} = \frac{\partial}{\partial t}\left[-v\frac{df(u)}{du}\right] = -v\frac{d^2 f(u)}{du^2}\frac{\partial u}{\partial t} = v^2\frac{d^2 f(u)}{du^2}.$$

This leads to

$$\boxed{\frac{\partial^2\psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi(x,t)}{\partial t^2}}$$

Note that when you square plus or minus v that you get positive v squared. This differential equation applies to waves moving to the left or to the right. This is the wave equation in one dimension. The general solution is a combination of a wave moving right and one moving left:

$$\psi(x,t) = Af(x - vt) + Bg(x + vt)$$

Just be careful. In practice it is best to use the forms from the last chapter, with the minus sign in front of the x for left-traveling waves per Perkins and Ruiz (2018).

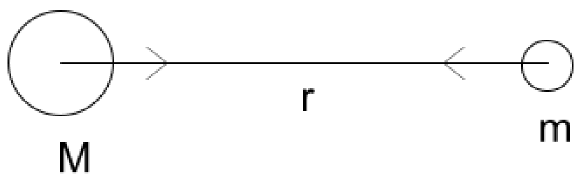
$$\psi_{right}(x,t) = Ae^{i(kx-\omega t)} \qquad \psi_{left}(x,t) = Ae^{-i(kx+\omega t)}$$

M1. Gauss's Law. We next need to develop the theoretical foundation for electromagnetic theory. You have seen the Maxwell equations in introductory physics where, they used the integral form for them. We review them now, starting with the first Maxwell equation, Gauss's Law.

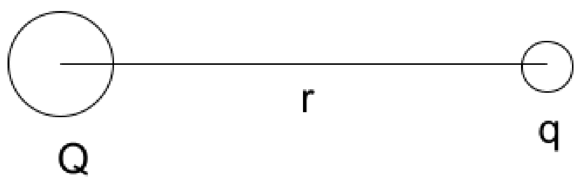
The first Maxwell equation is a restatement of Coulomb's Law in a form we call Gauss's Law. Coulomb's Law is

$$F_E = \frac{kQq}{r^2} \quad \text{where the constant} \quad k = \frac{1}{4\pi\varepsilon_0}$$

in the Meter-Kilogram-Second system of units. This form of the Metric System is often called the MKS system for short. The constant ε_0 is called the **permittivity of free space**. Note the similarity with the form of Newton's Law of Universal Gravitation below.



$$F_G = \frac{GMm}{r^2}$$



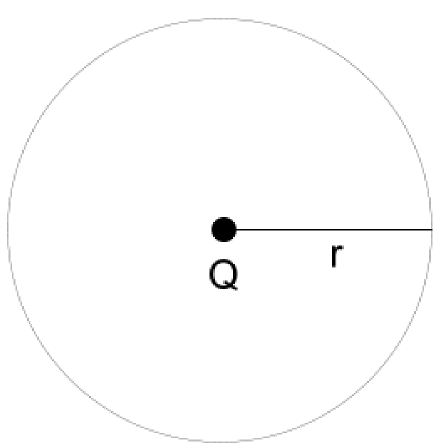
$$F_E = \frac{kQq}{r^2}$$

Both are inverse square laws. The Q represents charge, replacing the M which represents mass. The r is the distance between the centers of each mass or charge. Sometimes you see a minus sign in front of the gravity equation to remind us that the force is attractive. With the charges, when they are opposite in sign, you get the attraction.

Think about these properties called mass and charge. A fundamental force in nature means we endow matter with a property that goes with the force. For gravity it is mass M (or m). For the electric force, it is charge Q (or q). If you have mass, you experience gravity. If you have charge, you experience the electric force.

We define force fields for gravitation and the electric force by taking the smaller mass m to be 1 and the smaller charge q to be 1. Then we have forces per unit mass or charge:

$$g = \frac{GM}{r^2} \qquad\qquad E = \frac{kQ}{r^2}$$



Consider a charge Q at the origin and make a sphere at distance r to surround this charge. The electric field at a distance r from the charge is

$$E = \frac{kQ}{r^2}$$

Since our force is a vector and will point outward for a positive test charge q, we write this in vector form as

$$\vec{E} = \frac{kQ}{r^2}\hat{r}.$$

The $\hat{r}$ vector is a unit vector pointing away from the charge Q. Its precise direction depends on where you are on the sphere. This feature is unlike your unit vectors in Cartesian coordinates, $\hat{i}$, $\hat{j}$, and $\hat{k}$, which always point in the same directions. We proceed to define a differential patch of area on the sphere and give it a unit vector direction outward. This approach is common practice with areas, i.e., to define area orientations with unit vectors perpendicular to the surfaces. When you come to think of this, that is the most convenient way to tell someone how to orient a plane piece of paper - by a unit vector perpendicular to the paper for the direction you want.

$$\overrightarrow{dA} = \hat{r}dA$$

Don't worry about the actual details for dA as we will not need explicit expressions. We will be talking at the most fundamental level for the most part. We want to take $\vec{E}\cdot d\vec{A}$ and integrate over all the area on the sphere. When we integrate over a closed area we include a nice loop to emphasize that our area encloses on itself:

$$\oiint \vec{E}\cdot\overrightarrow{dA}$$

Let's do this integral. We have $\vec{E} = \frac{kQ}{r^2}\hat{r}$ and $\overrightarrow{dA} = \hat{r}dA$. Then,

$$\oiint \vec{E}\cdot\overrightarrow{dA} = \oint \frac{kQ}{r^2}\hat{r}\cdot\hat{r}dA.$$

The dot product $\hat{r}\cdot\hat{r}$ is equal to 1 since a dot product of any unit vector with itself is 1. The dot product between two vectors is the multiplication of the magnitudes times the cosine of the angle between them. The angle between a vector and itself is zero.

$$\oiint \vec{E}\cdot\overrightarrow{dA} = \oint \frac{kQ}{r^2}dA$$

Since we have a sphere here with a radius r that does not change and we want the surface area, we can pull out the constants:

$$\oiint \vec{E} \cdot \vec{dA} = \frac{kQ}{r^2} \oint dA \, .$$

This equation might still look scary but the integral is simply the surface area of a sphere. You know this. It is $A = 4\pi r^2$. So we get

$$\oiint \vec{E} \cdot \vec{dA} = \frac{kQ}{r^2} 4\pi r^2 = 4\pi kQ \, .$$

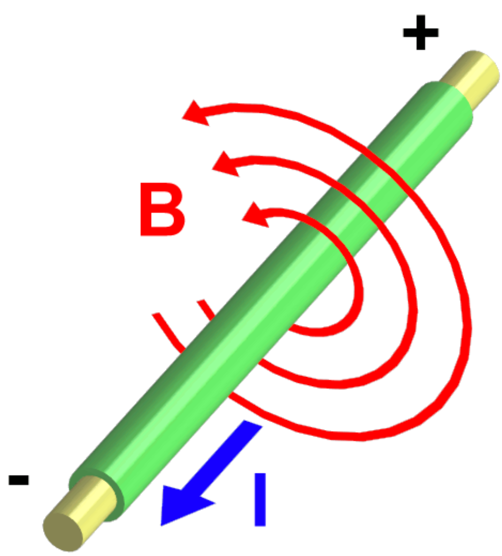
Note that our constant $k = \dfrac{1}{4\pi\varepsilon_0}$. So we wind up with

$$\oiint \vec{E} \cdot \vec{dA} = 4\pi kQ = 4\pi \frac{1}{4\pi\varepsilon_0} Q = \frac{Q}{\varepsilon_0}$$

This form is called Gauss's Law and it is our first Maxwell equation.

$$\boxed{\oiint \vec{E} \cdot \vec{dA} = \frac{Q}{\varepsilon_0}}$$

M2. The Magnetic Field. A current in a wire produces a magnetic field (Figure Courtesy Wikimedia: Wapcaplet, Creative Commons License). Due to the cylindrical symmetry, we can assign the unit vector $\hat{\theta}$. To get a sense of this direction, use your right hand with thumb aligned with the current. The B field then takes on the direction of your curved fingers.



$$\vec{B} = \frac{\mu_0 i}{2\pi r} \hat{\theta} \, .$$

You have seen the magnetic field B as a loop line integral wrapping around the wire from which you get the above result. Let's do this backwards. We then arrive at Ampère's Law in integral form.

$$B = \frac{\mu_0 i}{2\pi r}, \qquad B(2\pi r) = \mu_0 i, \quad \text{and} \quad \oint \vec{B} \cdot \vec{dl} = \mu_0 i \, .$$

We emphasize that both Coulomb's electrical law and the magnetic law are fundamental force laws like gravity. These laws are foundations – starting points. However, one can derive the magnetic force law using Coulomb's law and Einstein's theory of special relativity! The derivation is long. If you are curious, I describe the derivation in a 10-minute YouTube video.

The above magnetic field loop integral is the integral we can use to calculate B for the current in the wire. You did this in your introductory physics class. Since the magnetic field lines wrap around on themselves, if we swallow up a magnet like we did for a charge in Gauss's Law, we get zero. There are no net piercings of the magnetic field lines outward through the surface. What field lines go in also leave the enclosed space.

$$\boxed{\oiint \vec{B}\cdot\overrightarrow{dA} = 0}$$

This equation is the 2nd Maxwell equation. Part of the 3rd Maxwell equation is Ampère's Law

$$\boxed{\oint \vec{B}\cdot\overrightarrow{dl} = \mu_0 i}.$$

You also learned in introductory physics how a charged particle with charge q responds to the electric and magnetic field. The general force law which includes both electric and magnetic fields is called the Lorentz force law, named after Lorentz of Lorentz transformation fame.

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

The three key scientists involved so far in discoveries leading to the Maxwell equations are shown below. Coulomb is decked out in his uniform.

Charles Augustin de Coulomb (1736-1806)	Johann Carl Friedrich Gauss (1777-1855)	André Marie Ampère (1775-1836)
		

Courtesy School of Mathematics and Statistics, University of St. Andrews, Scotland

M3. Faraday's Law.

Michael Faraday (1791-1867)
Courtesy School of Mathematics and Statistics
University of St. Andrews, Scotland

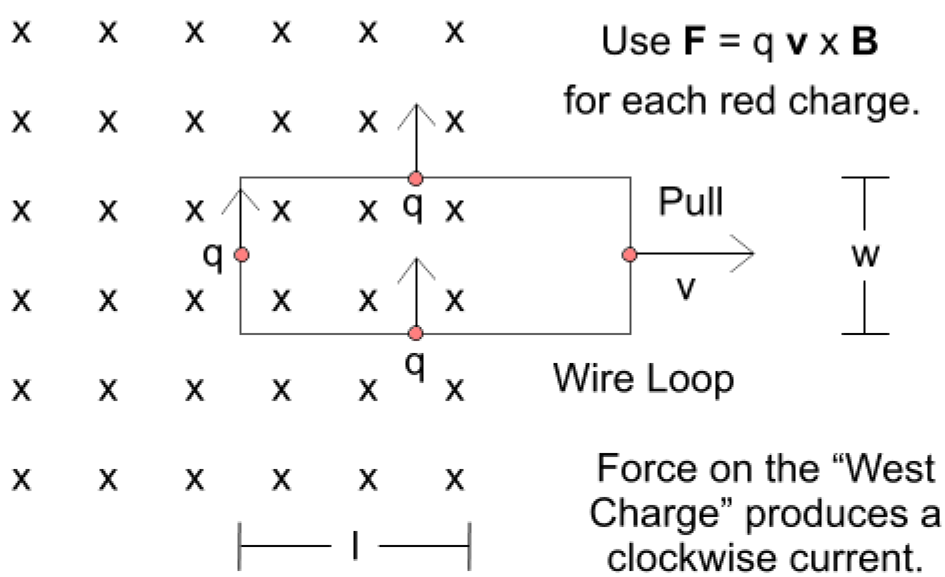
Here is Faraday's law, which you encountered in intro physics.

$$\boxed{\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}},$$

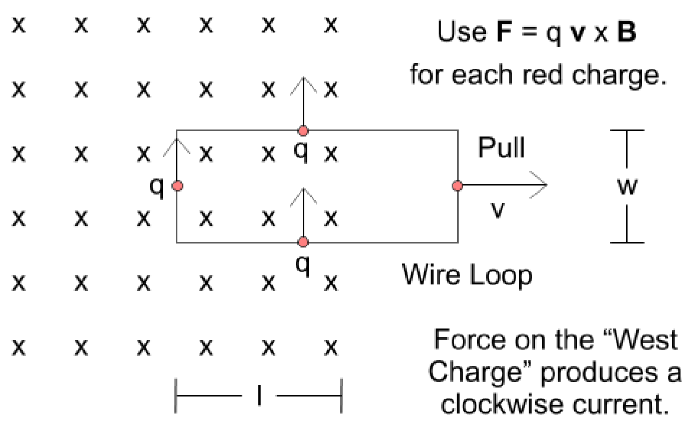
where Φ_B is the magnetic flux. Magnetic flux is found by multiplying the magnetic field with the area through which the field lines penetrate.

$$\Phi_B = BA$$

If the magnetic field is not constant, you have to do an integral. Let's see if we can understand a theoretical argument as to why Faraday's law is true. We start from what we know. The "x" marks below are the tails of the constant magnetic field B lines that goes into the page. We pull a wire loop through this field.



Apply $\vec{F} = q\vec{v} \times \vec{B}$ to each of the four red positive charges in the wire. These is no force on the east charge since B = 0 there. The other charges are pulled upward but only the west charge starts to move to produce a current due to $F = qvB$. An electric field is generated since we have induced current. The electric field generated must be $E = vB$ from the electric-force part of the Lorentz force law: $\vec{F}_e = q\vec{E}$. Therefore $qvB = qE$.



The velocity seen in our formula

$$E = vB$$

is given by $v = -\dfrac{dl}{dt}$, which is the negative of the decrease in our length portion where there is the magnetic field. These relationships lead to

$$E = vB = -\frac{dl}{dt}B \, .$$

Now consider the loop integral for this generated E field. The only relevant side is the west side.

$$\oint \vec{E} \cdot \vec{dl} = Ew \, .$$

On the west side the electric field lines up with the differential vector length element. We integrate along the path where the electric field is pointing. So we integrate up and therefore get the positive Ew. We should integrate counterclockwise since when we point out right thumb in the direction of the **B** field, our fingers curve counterclockwise.

The integral for the top part is zero since the E field is perpendicular to the direction which at the top is to the right. There is no E field on the east side. The integral at the bottom is zero similar to the top analysis.

Putting it all together, we obtain

$$\oint \vec{E} \cdot \vec{dl} = Ew = -\frac{dl}{dt}Bw \, .$$

To allow for pulling the wire upwards, we move w into the derivative. The w is constant here but would not be if we pulled upwards instead of the right.

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d(lw)}{dt} B \; .$$

Note that the product lw is the length times width for where the magnetic field penetrates through the wire. So we call this the area A = lw and write

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d(lw)}{dt} B = -\frac{dA}{dt} B \; .$$

Since the B is constant we can pull the B into the derivative. But this is significant because if we increased the B field instead of moving the wire, we would get the same effect.

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d(BA)}{dt} \; .$$

Since $\Phi_B = BA$, the magnetic flux, we have arrived at Faraday's Law from a theoretical analysis.

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$$

Now we have four equations, but we have to add one important piece. We do that in the next section.

$$\oiint \vec{E} \cdot \vec{dA} = \frac{Q}{\varepsilon_0}$$

$$\oiint \vec{B} \cdot \vec{dA} = 0$$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

M4. Maxwell's Equations.

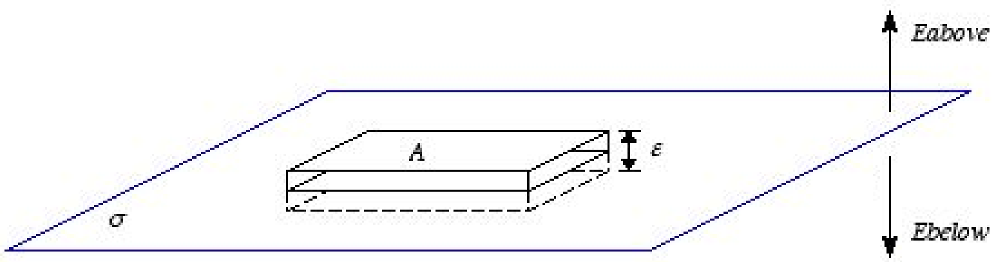
James Clerk Maxwell (1831-1879)
Courtesy School of Mathematics and Statistics
University of St. Andrews, Scotland

Maxwell focused on the fact that a changing magnetic flux produces an electric field. For cases when the magnetic field strength increases or decreases through a given area one can say that a changing magnetic field produces an electric field.

Could the reverse be true? Could a changing electric field produce a magnetic field? Could a changing electric flux mean we get a magnetic field?

He found that to be the case, which made possible electromagnetic waves, which we study later.

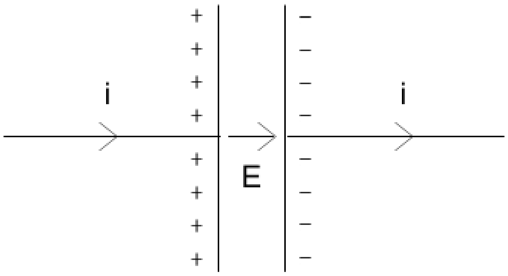
Before we analyze Maxwell's hunch, we will need a calculation you did in your intro physics course with Gauss's Law. To be complete, we will repeat it here. The problem is to find the electric field due to an infinite sheet of charge with density σ per unit area. We make a little rectangle box to enclose a piece of the plane inside.



Courtesy Prof. Frank L. H. Wolfs, Department of Physics
and Astronomy, University of Rochester, NY

We apply Gauss's Law $\oiint \vec{E} \cdot \vec{dA} = \frac{Q}{\varepsilon_0}$, where Q is the charge inside. The electric field is upward on the above surface and downward on the below surface. The result is

$$EA + EA = \frac{\sigma A}{\varepsilon_0} \text{, which gives } E = \frac{\sigma}{2\varepsilon_0} .$$



Earlier we worked out the magnetic field a distance from a wire with current i using Ampère's law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i \text{ and found } B = \frac{\mu_0 i}{2\pi r}.$$

What happens if we interrupt the current by placing two plates each with area A in the path? There is no current across the gap. Do we get a B outside the plate region? The two parallel plates make up a capacitor, a circuit element that can store charge. Those plates are getting charged up and the plates stop the current from going across the gap in the middle.

The electric field inside is the sum of two sheets of charge. We neglect the edge effects. Since each sheet produces $E = \frac{\sigma}{2\varepsilon_0}$ and the opposite charges on each side work together to produce an even stronger electric field, the total strength due to both sheets is $E = \frac{\sigma}{\varepsilon_0}$, i.e., double. The charge density on each plate is $\sigma = \frac{Q}{A}$, where Q is the absolute magnitude of the total charge on each plate and the area of each plate is A. In the spirit of Maxwell's insight, we calculate the change in electric flux between the plates.

$$\frac{d\Phi_E}{dt} = \frac{d(EA)}{dt} = \frac{d}{dt}\left[\frac{\sigma}{\varepsilon_0} A\right] = \frac{d}{dt}\left[\frac{Q}{\varepsilon_0}\right] = \frac{i}{\varepsilon_0}.$$

We make the bold statement that the magnetic field B should be the same outside the plates too. Then we need $\oint \vec{B} \cdot \vec{dl} = \mu_0 i$ here too. But there is no actual I in the plate region. Instead we have an effective current inside the plate via $\frac{d\Phi_E}{dt} = \frac{i}{\varepsilon_0}$, i.e., $i = \varepsilon_0 \frac{d\Phi_E}{dt}$. So we need add this effective current to the regular current in Ampère's Law:

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}$$

And this last piece of the puzzle will allow for the existence of light!

The Maxwell Equations and the Lorentz Force Law

$$\oiint \vec{E} \cdot \overrightarrow{dA} = \frac{Q}{\varepsilon_0}$$

$$\oiint \vec{B} \cdot \overrightarrow{dA} = 0$$

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d\Phi_B}{dt}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

M5. Electromagnetic Waves. We are interested in the vacuum, away from source charges and currents. The charges and currents are then far away. But there are electric and magnetic fields in the space we are in, caused by those far away sources. So we set the charges and currents to zero where we are and have the free space Maxwell equations.

Free Space Equations

$$\oiint \vec{E} \cdot \overrightarrow{dA} = 0$$

$$\oiint \vec{B} \cdot \overrightarrow{dA} = 0$$

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d\Phi_B}{dt}$$

For the free space Maxwell equations we are far away from any charge sources and currents. Thus, we have set

$Q = 0$ and $i = 0$.

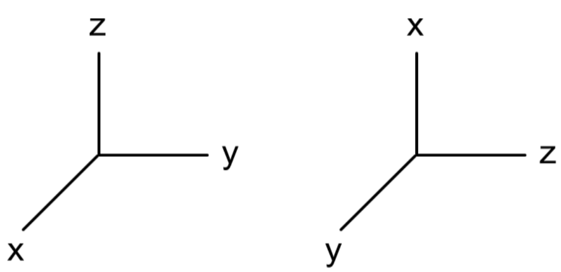
The free-space equations have beautiful symmetry and contain the secret about light. We play with these equations to see if a wave equation is supported. This seeking is an example of theoretical physics at its best. We are in search of a discovery starting with the Maxwell equations and using theory only from this point onward.

The Secret to Understanding Light

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$$

What might inspire us? Studying the meaning of equations by staring at them is very valuable. The first equation tells us that a changing electric flux produces a magnetic field. The second indicates that a changing magnetic flux produces an electric field. Could a changing electric field then create a changing magnetic field that in turn creates a further electric field and so on? Perhaps we get a sort of chain reaction and the fields propagate themselves. And we are in vacuum! That would mean electromagnetic waves need no medium to travel in or on! There would then be **no ether**, once thought to support waves in an otherwise vacuum.

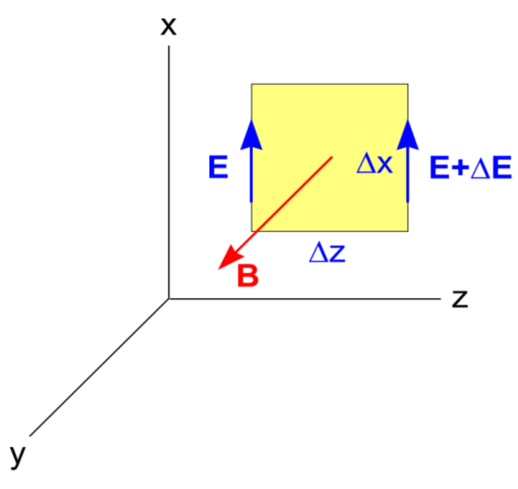


We first set up a coordinate system. I prefer to turn the usual coordinate system (left one) to the orientation seen in the right coordinate system.

Note that I have respected the original. This respect is important since both systems remain as right-handed coordinate systems, i.e.,

$$\hat{i} \times \hat{j} = \hat{k} \, .$$

To start things off, we take an electric field along the x axis and a B field along the y axis. For the loop integral in Faraday's law we integrate counterclockwise since when we point our right thumb along the B field, our fingers curve counterclockwise.



$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{E} \cdot \vec{dl} = (E + \Delta E)\Delta x - E\Delta x = \Delta E \Delta x$$

$$\Delta E = \frac{\partial E}{\partial z} \Delta z$$

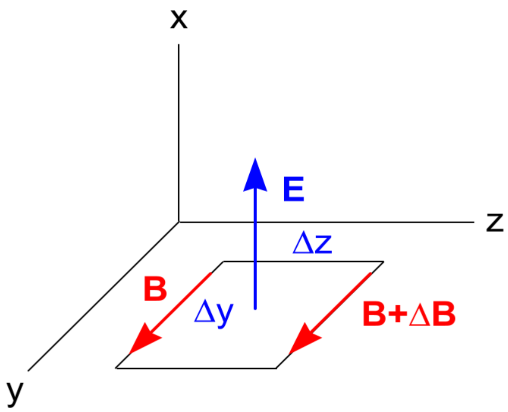
$$\oint \vec{E} \cdot \vec{dl} = \frac{\partial E}{\partial z} \Delta z \Delta x$$

$$-\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\Delta z \Delta x) = -\frac{\partial B}{\partial t} \Delta z \Delta x$$

$$\boxed{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}}$$

Next we consider $\oint \vec{B} \cdot \vec{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$.

We again integrate counterclockwise, since the electric field points upward.



$$\oint \vec{B} \cdot \vec{dl} = B\Delta y - (B + \Delta B)\Delta y = -\Delta B \Delta y$$

$$\oint \vec{B} \cdot \vec{dl} = -\frac{\partial B}{\partial z} \Delta z \Delta y$$

$$\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \varepsilon_0 \frac{d}{dt}(E \Delta z \Delta y)$$

$$\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \Delta z \Delta y$$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\boxed{-\frac{\partial B_y}{\partial z} = \mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t}}$$

The two derived equations are $\boxed{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}}$ and $\boxed{-\frac{\partial B_y}{\partial z} = \mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t}}$.

Take another derivative since we are shooting for the wave equation.

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial z} \frac{\partial B_y}{\partial t} = -\frac{\partial}{\partial t} \frac{\partial B_y}{\partial z} = \frac{\partial}{\partial t}\left[\mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t}\right]$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

WAVE EQUATION! E Traveling down z-axis.
Take another derivative on the second equation.

$$-\frac{\partial^2 B_y}{\partial z^2} = \frac{\partial}{\partial z}\left[\mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t}\right] = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\frac{\partial E_x}{\partial z} = \mu_0 \varepsilon \frac{\partial}{\partial t}\left[-\frac{\partial B_y}{\partial t}\right]$$

$$\frac{\partial^2 B_y}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$

WAVE EQUATION! B also traveling down z-axis.

LIGHT IS A TRANSVERSE WAVE, in both the E and B Fields.

They are perpendicular to the direction of propagation (thus **transverse wave**).

Light also needs no medium to travel in. It propagates itself via the Maxwell equations.

The **E** vector is vertically aligned. We say the light has **linear polarization**, in this case vertical.

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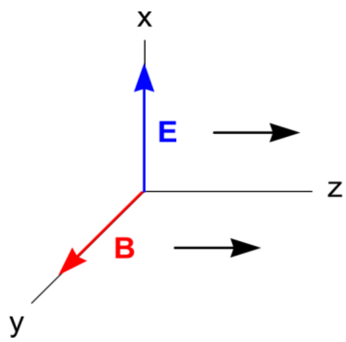


$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \qquad \frac{\partial^2 B_y}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$

Compare with

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi(x,t)}{\partial t^2},$$

The wave speed is

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Time to plug in some values. We will use current day values.

Electric constant: $\varepsilon_0 = 8.8541878128 \times 10^{-12} \ \frac{\mathrm{C}^2}{N \cdot m^2}$

Magnetic constant: $\mu_0 = 1.25663706212 \times 10^{-6} \ \frac{T \cdot m}{A}$

$$v = \frac{1}{\sqrt{(1.25663706212 \times 10^{-6})(8.8541878128 \times 10^{-12})}} \frac{m}{s}$$

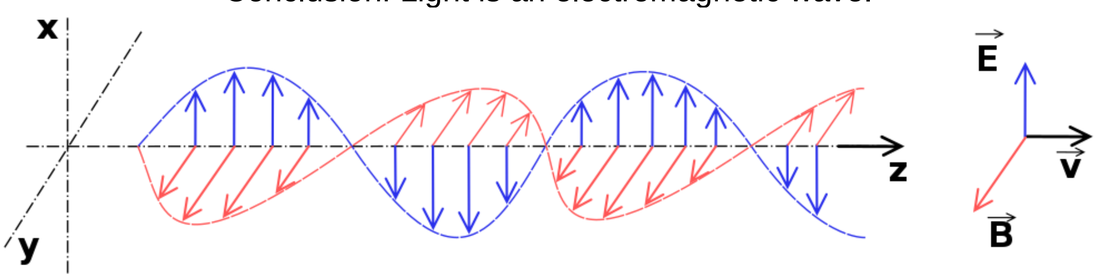
$$v = 299{,}792{,}458 \, \frac{\mathrm{m}}{\mathrm{s}}$$

The speed of light by checking the references is

$$c = 299{,}792{,}458 \, \frac{\mathrm{m}}{\mathrm{s}}$$

Not likely a coincidence finding agreement to 9 significant figures!

Conclusion: Light is an electromagnetic wave.



Representation of Light Wave. Wikipedia: SuperManu. [Creative Commons License]

$$c = 299{,}792{,}458 \, \frac{\mathrm{m}}{\mathrm{s}} = 299{,}792.458 \, \frac{\mathrm{km}}{\mathrm{s}}$$

Let’s do some rounding off using kilometers per second for the speed of light.

$$c = 299,792 \frac{\text{km}}{\text{s}}$$ (6 significant figures)

$$c = 299,790 \frac{\text{km}}{\text{s}}$$ (5 significant figures)

$$c = 299,800 \frac{\text{km}}{\text{s}}$$ (4 significant figures)

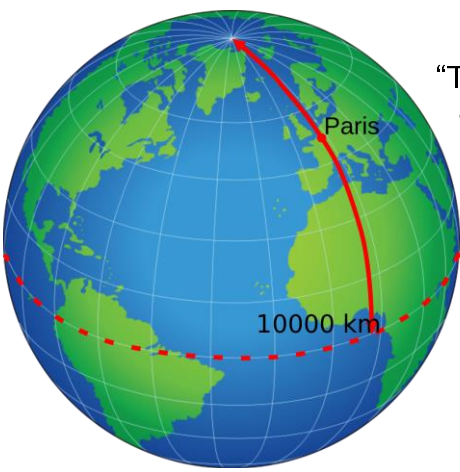
$$c = 300,000 \frac{\text{km}}{\text{s}}$$ (? significant figures)

The above value is really to 3 significant figures. To communicate this fact, we can write

$$c = 3.00 \times 10^6 \frac{\text{km}}{\text{s}}.$$

In 1983, an international agreement was made to tweak the definition of the meter so that the speed of light is EXACTLY $c = 299,792,458 \frac{\text{m}}{\text{s}}$.

The meter is defined as the distance light travels in vacuum in 1/299,792,458 second. We have come long ways since the days of that one-meter reference bar in France.



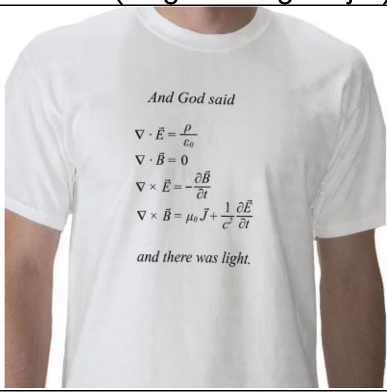
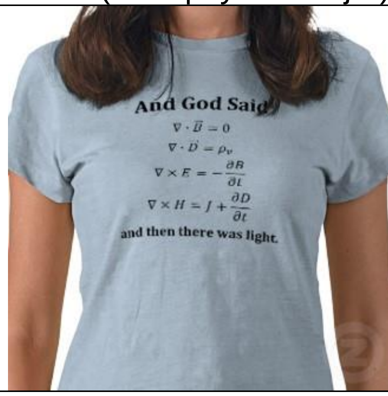
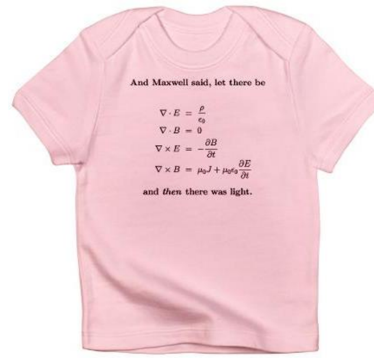
"The metre was originally defined in 1791 as being 1/10,000,000 of the distance from the North Pole to the Equator through Paris, making the kilometer 1/10,000 of this distance." Wikimedia

Public Domain image from Wikimedia: adapted by Martinvl

Original was prepared by a US Government employee during employment and is thus Public Domain.

"Let There Be Light."

T-Shirts using the differential form for the Maxwell equations in various versions.

<table>
<tr><td>Brother (Engineering Major)</td><td>Sister (Astrophysics Major)</td><td>Baby (Undecided)</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td colspan="2">www.zazzle.com for Custom T-Shirts</td><td>www.cafepress.com</td></tr>
</table>

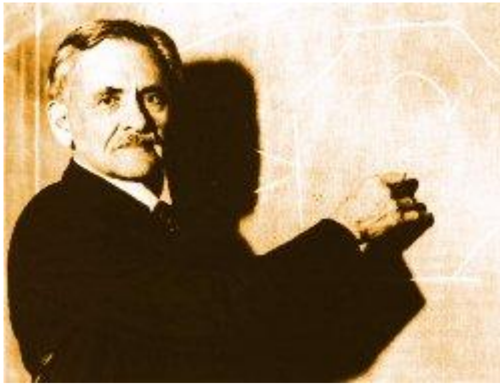
I prefer the MKS units on the left shirt, which agrees with the right box below.

The Maxwell Equations in Integral Form (left) and Differential Form (right).

$$\oiint \vec{E} \cdot \vec{dA} = \frac{Q}{\varepsilon_0}$$
$$\oiint \vec{B} \cdot \vec{dA} = 0$$
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$
$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

M6. Measuring the Speed of Light.



Albert Michelson was born in Poland. He came to the United States and attended the ***US Naval Academy*** in Maryland. He stayed on as a teacher. In 1878, with $10 worth of equipment, he set up an experiment to measure the speed of light using a rotating 8-sided mirror and a regular plane mirror on a mountain 22 miles away (along the seawall).

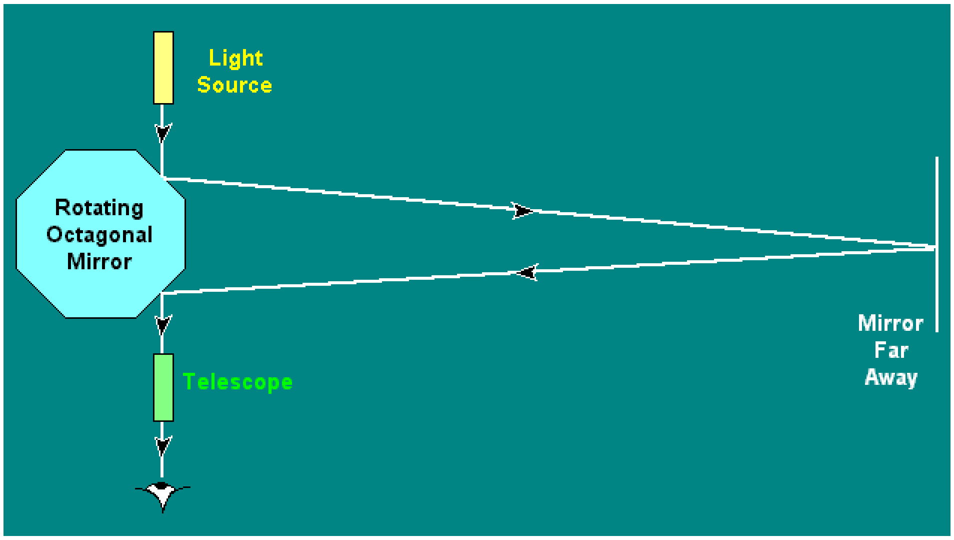
In the photo, Michelson is at the blackboard with a sketch of his 8-sided mirror. This experimental result was the first accurate measurement of the speed of light. Albert Michelson became the first American to win the ***Nobel Prize in Physics*** (1907). Your instructor had the good fortune to meet one of Michelson's daughters years ago.

"I had the pleasure of meeting one of Michelson's daughters, Dorothy Michelson Livingston, shortly after I started teaching at ***UNCA***. Livingston came to ***UNC-Greensboro***, North Carolina in 1979 to talk about her father and a book she had written about him. The occasion was a regional meeting of the ***American Association of Physics Teachers***. She told captivating personal stories, mentioning Einstein's visits to her home to see her father when she was a little girl. After her talk, there followed a presented-paper session in which I gave a talk on the ***Light*** course I was developing at ***UNCA***. My presentation included a slide show of images I had collected using all major parts of the electromagnetic spectrum. I was glad to see that Michelson's daughter was in the audience for my presentation. During my introductory remarks I mentioned that all electromagnetic waves travel at the speed of light, accurately measured by Michelson. I moved my arm to acknowledge Livingston when I mentioned her father. During the break that followed, Livingston quickly came down to the front of the room excited about my presentation and smiling. She said she would take my course if it were offered where she lived in New York." ***Prof. Ruiz, January 2001 Recalling a 1979 Experience***. She could now online!

Here is a description of Michelson's famous experiment to accurately measure the speed of light. The light hits the top right mirror of the octagonal mirror (see figure). The light bounces off the mirror and heads to the plane mirror on a mountain 22 miles away. It reflects off the mountain and travels 22 miles back. But by this time, Michelson's mirror has rotated 1/8 of a turn so that the returning light can hit the lower right mirror and bounce into the telescope at the bottom of the figure. If the octagonal mirror is too slow, the light will not reflect in the correct direction and thereby miss the telescope.

Michelson had to rotate his mirror 535 times per second. To make the calculation easy to do without a calculator, we are going to round off 535 to **500 Hz**. Now if we actually did the experiment by spinning the mirror slower at 500 Hz, we would have to use a mountain farther away since light has slightly more time to travel in our revised experiment. In our revised experiment, the light would have to travel a round trip of **75 km**. This is about 47 miles instead of Michelson's 44 miles for the round trip of going 22 miles each way.

Michelson's Experiment to Measure the Speed of Light



I use rounded values for my *PHYS 101 Light and Visual Phenomena* class so we can concentrate on the physics with easier numbers to work with. And after all the Maxwell equation stuff we did today, you need a break from heavy math and numbers. Here is my basic math approach that you can use for the layperson and grade-schoolers.

- Distance for round trip = 75 km,
- Time for round trip = (1/8)(1/500) s = 1/4000 s

We are ready to figure out the speed using our standard relation

speed = distance / time.

Since we are going to divide by a fraction, we use the prescription: keep, change, invert. We keep the 75, change division to multiplication, and invert the fraction to get 4000:

speed = 75 x 4000 = 25 x 3 x 4000 = 25 x 4 x 3000 = 300,000 km/s.

This value of ***300,000 kilometers per second*** is about ***186,000 miles per second***. If you could circle the Earth at this speed, you would whip around the Earth over 7 and a half times in one second since the circumference of the Earth is 25,000 miles.

A student from ***UNCA*** wrote the following letter to your physics teacher in the early 1980s after he took the light course and spent some time at the ***U.S. Naval Academy***.

Dr. Ruiz,

Doubt you remember me but I was in a few of your classes last year before I transferred here. I was wandering around the campus when I came across a small plaque that I thought might be an interesting bit of trivia for you. In your "light" course you talked about the speed of light. This plaque had something to do with it.

"Midshipman Albert A. Michelson graduated from the Naval Academy in 1873, but stayed on to teach physics and chemistry. using that time he set up on the Old North Seawall an apparatus for measuring the speed of light, and was the first man to accomplish this accurately ..."

Since the time of the experiment much of the Severn River and Annapolis Harbour have been filled in to provide more land for the academy. The sight of his experiments now runs across the center of the Yard. Tracing the path of his experiment is a small line of brass plates embedded in the ground. Also, inside the Physics and Math buildings are a couple cases containing the machines and equipment he used in his experiments along with many of his awards and prizes.

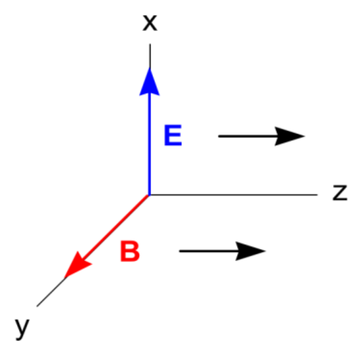
Just thought you'd be interested.

A former student,
Kevin J. Fitzpatrick
"Vicious"

Of course your physics teacher remembered Kevin. After Kevin wrote this letter, Kevin's brother, wife, and father all took ***Light and Visual Phenomena***.

A Plaque at the United States Naval Academy (USNA). Courtesy USNA.

M7. Light Wave.



$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$ $$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$ $$\frac{\partial^2 B_y}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$ $$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$$

$$E_x = E_o \sin(kz - \omega t) \qquad B_y = B_o \sin(kz - \omega t + \phi)$$

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t} \quad \Rightarrow \quad \frac{\partial}{\partial z} E_o \sin(kz - \omega t) = -\frac{\partial}{\partial t} B_o \sin(kz - \omega t + \phi)$$

$$kE_o \cos(kz - \omega t) = \omega B_o \cos(kz - \omega t + \phi)$$

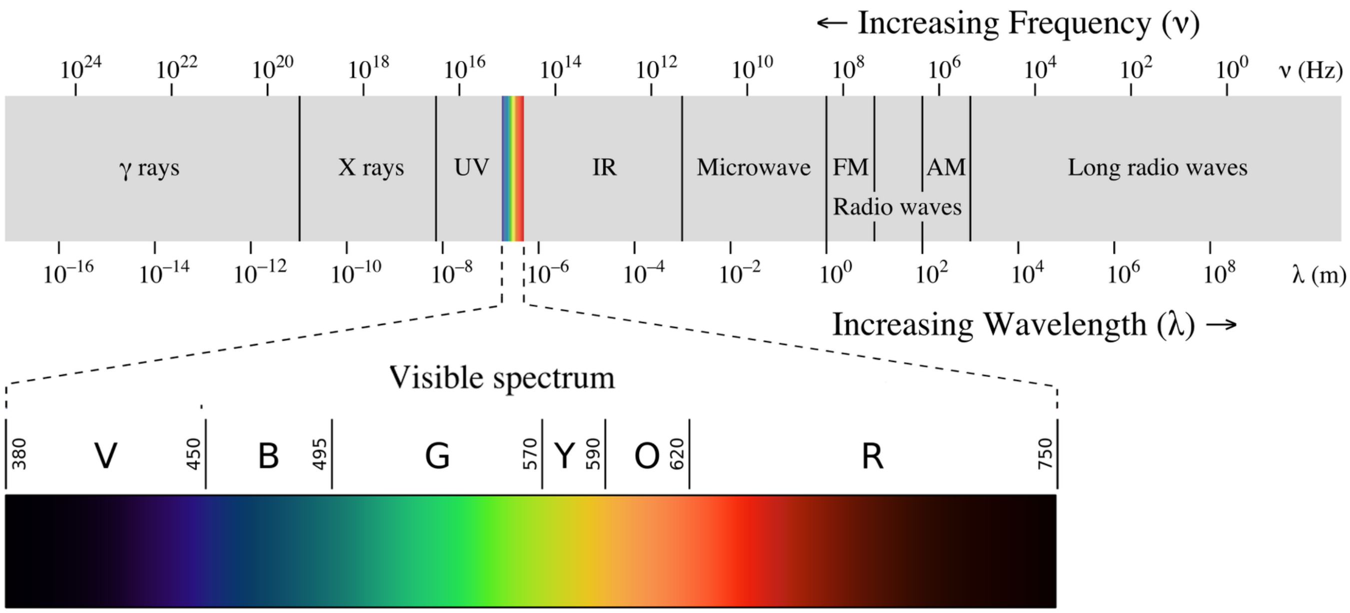
$$kE_o = \omega B_o \qquad \phi = 0$$

$$E_o = \frac{\omega}{k} B_o = cB_o$$

$$\frac{E_o}{B_o} = c$$

$$E_x = E_o \sin(kz - \omega t) \qquad B_y = B_o \sin(kz - \omega t)$$

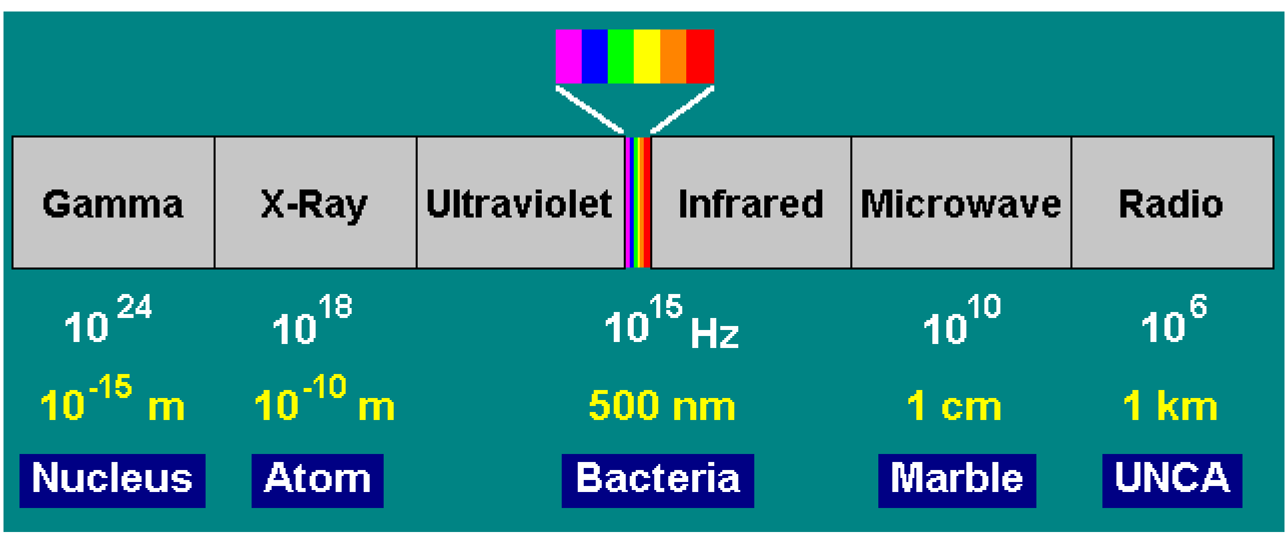
M8. The Electromagnetic Spectrum.



EM Spectrum. Wikipedia: Philip Ronan, Gringer. [Creative Commons License]

$$\boxed{c = \lambda f}$$

The Electromagnetic Spectrum



Chemistry Prof. Debbie Gale Mitchell Uses Makeup to Teach the Spectrum

Twitter, Friday, January 26, 2018.
Used by Permission
Courtesy Prof. Debbie Mitchell
Chemistry, University of Denver

"This chemistry prof's makeup is inspired by spectroscopy. Debbie Gale Mitchell ordered the colours of her eyeshadow by wavelength to teach her students about spectroscopy, which is the study of how light interacts with matter." Style - **Twitter**

"Hi, I'm a chemistry prof who loves over-the-top makeup. I’m teaching spectroscopy today, so 🌈 seemed appropriate. Colors are ordered by wavelength (λ). Longer λ was placed in the inner corner with λ decreasing towards outer corner." Prof. Mitchell

Prof. Debbie Gale Mitchell, Department of Chemistry and Biochemistry, University of Denver, Denver, Colorado (Ph.D. from U. of Denver in 2013)

The Professor had 4 basic colors of makeup, which she posted as hearts.

💛💚💙💜

Look at the photo’s left eye. The order from left to right is VBGY from the VBGYOR order.
Look at the photo’s right eye. The order from left to right is YGBV from the ROYGBIV order.

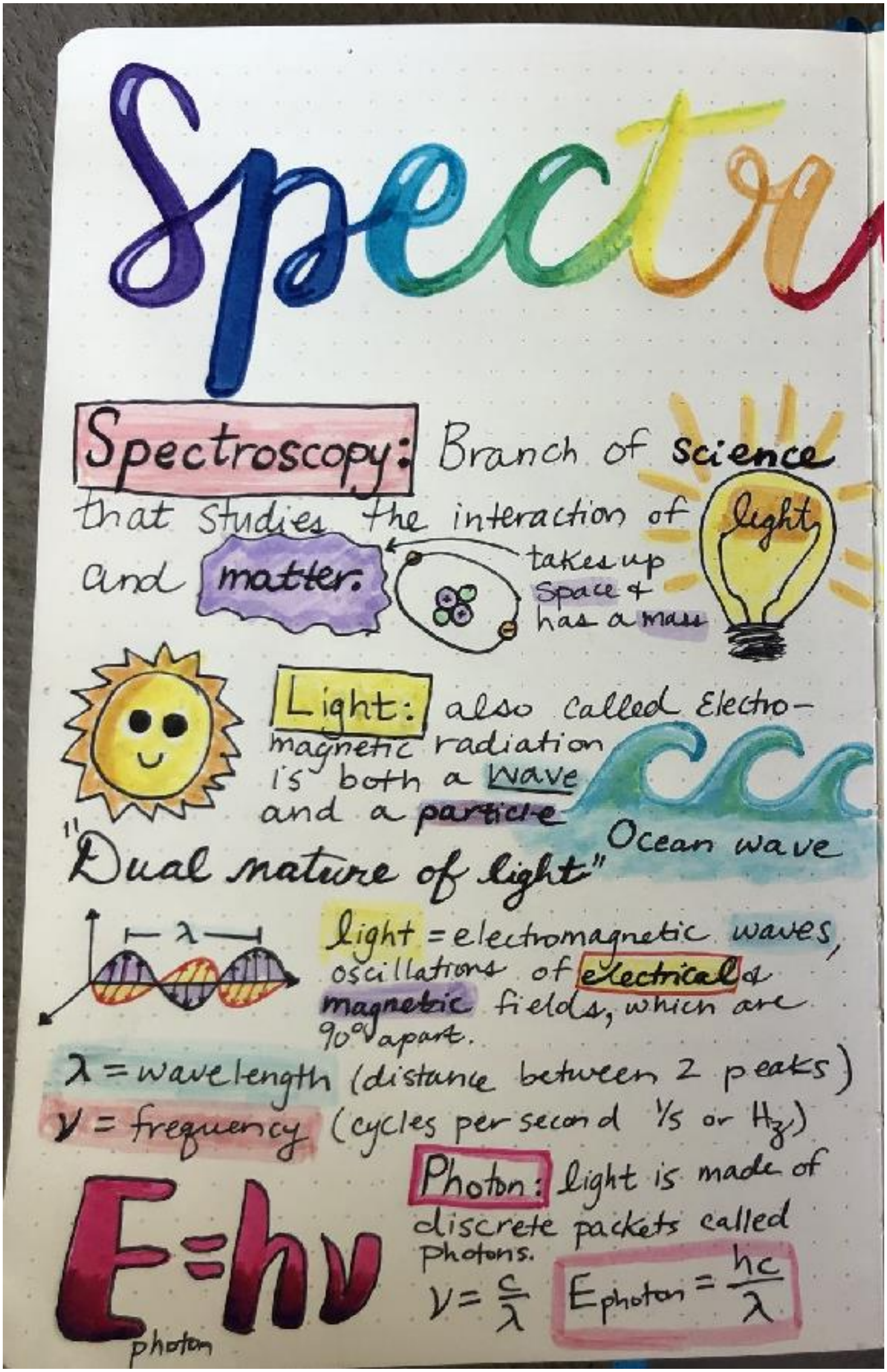
Sometimes scientists like to use the ROYGBIV order since energy increases from Red to Violet. Other times, scientists may use VBGYOR, where the wavelength increases as you go from Violet to Red. What is especially nice about Prof. Mitchell’s make-up idea is that there is a mirror symmetry in the applied make-up color array. She starts closest to the nose with the lower energy (longer wavelength) and proceeds outward for higher energies (shorter wavelength). The mirror reflection is an extra optical bonus.

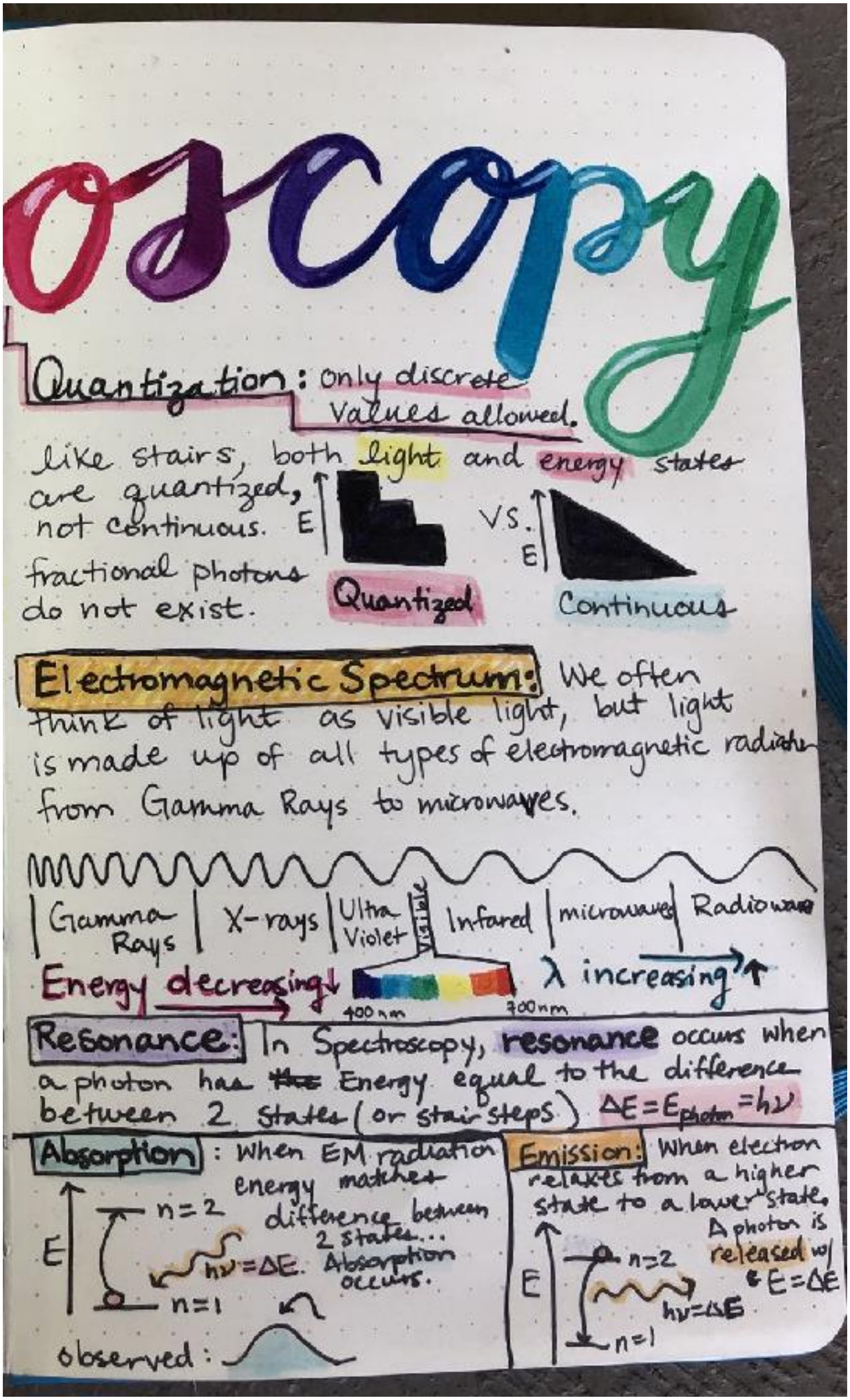
From Prof. Debbie Gale Mitchell on Twitter: “Spectroscopy notes, just for fun!”

Used by Permission, Courtesy Professor Debbie Gale Mitchell, Chemistry, University of Denver.

There is a lot of physics and chemistry on these pages. Remember our theme in the last chapter where we said it is important for you to eventually become a tour guide for landscapes of physics? Then you can tell the story of what you have learned in your own creative way. Chemistry Professor Mitchell sets a great example here laying out a landscape rich in both physics and chemistry. Can you find where she includes electromagnetic waves? She indicates that the electrical and magnetic fields are 90° apart as we found in this chapter.

The separate pages appear below.


Spectr
Spectroscopy: Branch of science that studies the interaction of light and matter.
takes up space & has a mass
Light: also called Electro-magnetic radiation is both a wave and a particle
Ocean wave
"Dual nature of light"
λ
light = electromagnetic waves, oscillations of electrical & magnetic fields, which are 90° apart.
λ = wavelength (distance between 2 peaks)
ν = frequency (cycles per second 1/s or Hz)
Photon: light is made of discrete packets called Photons.
E = hν
photon
ν = c/λ
Ephoton = hc/λ


oscopy
Quantization: only discrete values allowed.
like stairs, both light and energy states are quantized, not continuous.
E
VS.
E
fractional photons do not exist.
Quantized
Continuous
Electromagnetic Spectrum: We often think of light as visible light, but light is made up of all types of electromagnetic radiation from Gamma Rays to microwaves.
Gamma Rays
X-rays
Ultra Violet
Visible
Infared
microwaves
Radiowaves
Energy decreasing
400 nm
700 nm
λ increasing
Resonance: In Spectroscopy, resonance occurs when a photon has the Energy equal to the difference between 2 states (or stair steps) ΔE=Ephoton=hν
Absorption: When EM radiation energy matches difference between 2 states... Absorption occurs.
n=2
E
hν=ΔE
n=1
observed:
Emission: When electron relaxes from a higher state to a lower state. A photon is released w/ E=ΔE
n=2
E
hν=ΔE
n=1