HW-B1. Index of Refraction for Air. In our text we stated that the index of refraction for air was n = 1. You may Google for a more precise value and find something like n = 1.00024. Both of these values are not good enough when we need the index of refraction at a cool and hot temperature to obtain the mirage angle $\alpha = \sqrt{2(1 - \frac{n_{\text{hot}}}{n_{\text{cold}}})}$. In other words, it is the subtle difference in index of refraction due to the different temperatures that causes the mirage. Using temperature data from my publication [Michael J. Ruiz, "Road Mirage Angle," *Physics Education* **54**, 065009 (November 2019)], the ambient weather temperature was $T_{\text{cold}} = 27.5\ ^\circ\text{C}\ \pm 1.0\ ^\circ\text{C}$ and the hot air near the road surface was taken to be $T_{\text{hot}} = 53.0\ ^\circ\text{C}\ \pm 3.0\ ^\circ\text{C}$. These temperatures are 82 °F and 127 °F respectively.

Much research has been done to arrive at empirical formulas for the dependence of air temperature on pressure, temperature, humidity, and wavelength. All is given in the classic paper Birch K P and Downs M J 1994 Correction to the updated Edlén equation for the refractive index of air *Metrologia* **31** 315-6. Luckily, we can neglect the dependence on humidity since that effect is orders of magnitude smaller than pressure and temperature. But what about wavelength? From the Birch and Downs paper, we can write an empirical formula for the index of refraction of air as

$$n = 1 + A(\lambda) \cdot 10^{-8} \frac{p}{T + 273.15}$$, where here T is in degrees Celsius, P is in pascals, and

$$\frac{A(\lambda)}{0.0028425} = 8342.54 + 2406147\left[130 - \frac{1}{\lambda_o^2}\right]^{-1} + 15998\left[38.9 - \frac{1}{\lambda_o^2}\right]^{-1}$$ with λ_o in μm.

a) Show that at 633 nm = 0.633 μm, the red wavelength for a helium-neon laser, the index of refraction for air is as given below. [6 points for neatly and clearly showing all steps.]

$$n_{633} = n(\lambda = 633 \text{ nm}) = 1 + \frac{78.6\,p}{T + 273.15} 10^{-8}$$

b) When I taught engineers statics and dynamics for 7 years at UNCA as an North Carolina State University Adjunct Professor of Mechanical and Aerospace Engineering, I discovered that engineering students are taught better how to arrive at precise numbers, while physics majors are taught better how to do derivations. Well, it is time for you to be the best of both worlds. Give the index of refraction formula for 455 nm, the wavelength for a handheld blue laser and that for 532 nm, the wavelength for a handheld green laser. [2 pts each for nailing the correct coefficient analogous to 78.6 for the 633 nm case, i.e., three significant figures.]

When I did my research on mirages, I could not find a collection of such coefficients anywhere. I just found the master formula above and a couple of specific wavelengths in books here and there. So your homework problem is a research problem. You will most likely not find the answer anywhere for the 455 nm and 532 nm cases. Of course, as with all homework, you are encouraged to work with your classmates as a team and write it up in your own words and style.

HW-B2. The Mirage Angle and Wavelength. In my mirage paper I had to worry if wavelength affected the mirage angle $\alpha = \sqrt{2(1 - \frac{n_{\text{hot}}}{n_{\text{cold}}})}$. You are going to check the sensitivity on wavelength by using index of refraction formulas for an extreme red and violet in the visible spectrum:

$$n(\lambda = 400\text{ nm}) = 1 + \frac{80.4\,p}{T + 273.15} 10^{-8} \quad \text{and} \quad n(\lambda = 700\text{ nm}) = 1 + \frac{78.4\,p}{T + 273.15} 10^{-8},$$

where T is in degrees Celsius (°C) and pressure p is in pascals (Pa).

The weather data for the day of observation was

Temperature Cold: $T_{\text{cold}} = 27.5\ ^\circ\text{C}$ (ambient air temperature)

Temperature Hot: $T_{\text{hot}} = 53.0\ ^\circ\text{C}$ (air temperature near hot road surface)

Pressure: was standard atmospheric pressure to 3 significant figures, $p = 1.01 \times 10^5\ \text{Pa}$

You need to first calculate $n_{\text{cold}}(400\text{ nm})$ and $n_{\text{hot}}(400\text{ nm})$ using

$$n(\lambda = 400\text{ nm}) = 1 + \frac{80.4\,p}{T + 273.15} 10^{-8}.$$

Then you proceed to $\alpha = \sqrt{2(1 - \frac{n_{\text{hot}}}{n_{\text{cold}}})}$ in order to find α_{400}. Then you do the same steps to find α_{700}. Express each of these angles in degrees to two significant figures. What can you conclude?

Grading Rubric and Rationale. 4 points for steps with neat work, 2 points all of nothing for each of the numerical results for the two angles, 2 points for conclusion. It is possible in previous physics classes you found less weight placed on the actual numerical answer in a homework problem. Such is often true for intro classes where demonstrating you know how to set things up is more important than the actual answer.

But some of our assignments have applied numerical results like in engineering. In such cases, in a job situation, it is extremely important to confidently give the correct answer with appropriate units. So for these types of questions, the numerical answers are weighted heavily. Again, you are encouraged to work with your fellow classmates (as an engineering team) whenever you do homework, as that is the usual practice in a job situation – as long as you write up your homework in your own math, words, and style.

Due to the unusual social distancing as this course is being offered, it might be difficult to work together like the “gang” hanging out in Robinson-Rhoades Hall 119, where I often see a blackboard filled with physics homework. Therefore, I make this offer for our course – you may come to Zoom Office hours and show me your work so far or email me your work and I can let you know if you are on the right track.