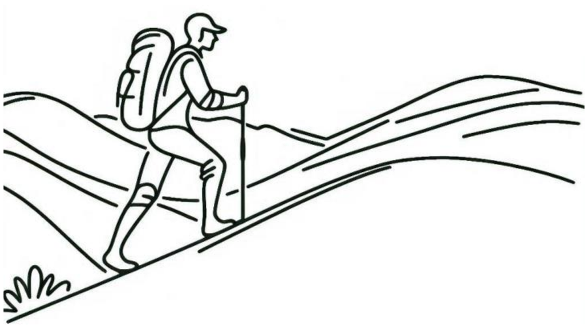
Basic Calculus. Prof. Ruiz (Doc), UNC-Asheville (1978-2021), DoctorPhys on YouTube.
Chapter 0. Topics. Prerequisites: High School Algebra and Trig.

This set of notes is aimed at students who want to learn the basics of calculus in preparation of more advanced studies in areas of physical science and engineering. Basic Calculus can also serve as a review of calculus for those who have studied calculus long ago. There is no attempt to be rigorous in the mathematical sense. Rigorous treatment represents the gold standard of calculus courses. Instead, our course is a practical one that introduces the fundamental concepts required for applications in physical science and engineering.

Physical science and engineering encompass broad areas. Subjects falling under the general heading of physical science include physics, astronomy, chemistry, and meteorology (atmospheric science). Another example is geodesy, which uses coordinate systems to describe the shape of the Earth and its gravity field. Engineering likewise has many subdivisions such as mechanical engineering, aerospace engineering, electrical engineering, civil engineering, and nuclear engineering.

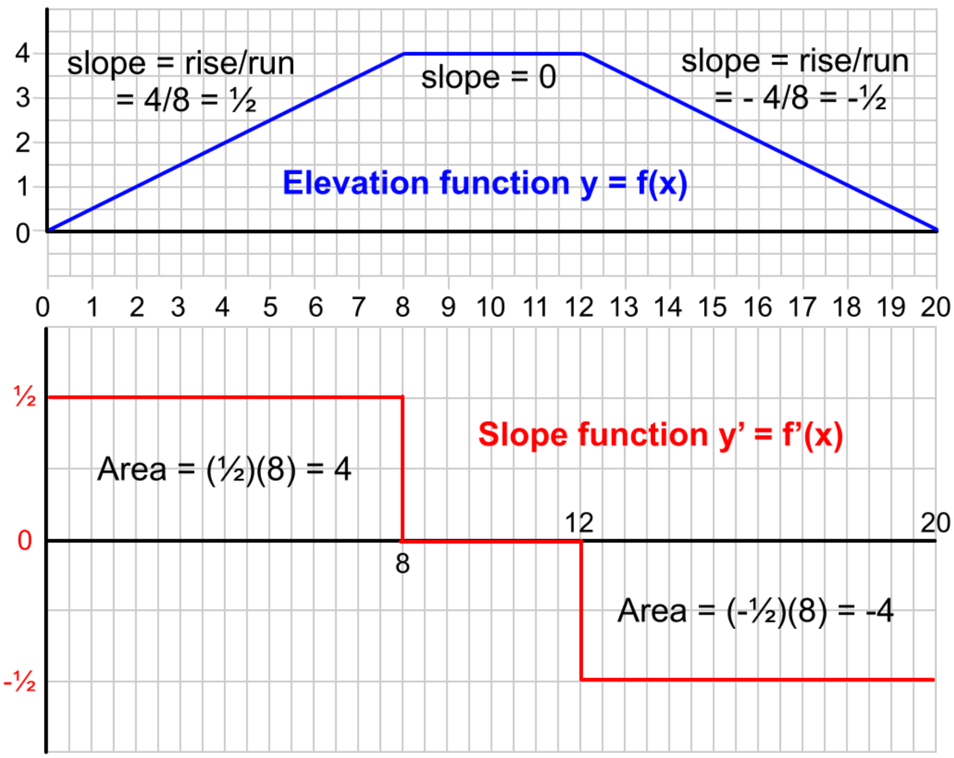
If you are a student entering these fields, I strongly recommend taking three semesters of college calculus with mathematicians and a fourth course in differential equations, also where the teacher is a mathematician. Mathematicians give rigorous treatment of calculus. My loose set of notes here can complement the rigor of the math courses by serving as a review or introduction to many of the basic concepts we need for applications.



I will explain the essence of differential calculus using a hiker on a trail where the trail requires hiking from left to right as shown in the figure. The contour of the ground indicates the elevation at each point. We need to choose a coordinate system to label each point on the trail. We will use the coordinate pair (x, y), where the origin $(x, y) = (0, 0)$ is chosen at the lower left of the figure where the trail begins. Elevation y will depend on x, i.e. for each value of x there will be an elevation y. We express this feature by writing $y = f(x)$. We call $f(x)$ a function, where f stands for function. You plug in a value for x and out comes an elevation $y = f(x)$. Since hiking trails do not have simple formulas that give the height in general, we can consider our function $y = f(x)$ as a table of values (x, y) stored in a computer. You give the computer a value for x and the computer gives you the corresponding elevation y.

The essence of differential calculus is to find the slope at each point for our trail. This new function is derived from the data of our elevation function $f(x)$. We call the new derived function $y' = f'(x)$ and we refer to it as the derivative, i.e., a derived function. The next major subheading of freshman calculus is integral calculus. The aim for integral calculus is to go backwards and find the antiderivative. In other words, you start with the slope function as given and then proceed to find the elevation function. Note that when going backwards, the slope function does not tell us where elevation zero is located as it only contains the slope information. So we need to add an overall constant when determining the antiderivative. It is good to know the elevation and the slope when hiking since very high elevations have less oxygen and steep slopes are hard to climb.

Below is an example of a simple idealized trail with numerical values. The elevation function is $y = f(x)$ and the slope function is $y' = f'(x)$. The slope is found by considering the "rise" over the "run," i.e., RISE/RUN. The elevation function shown below can be broken into three parts: a region with an upward slope, then a portion that is flat (zero slope), and finally a section that slopes downward (negative slope). The first section has a run of 8 and rise of 4 giving a slope of ½. The slope in the middle section is 0, and the third region drops down 4 units with a run of 8 units, giving a slope equal to -1/2. Note that if we measured distances in meters, the units would cancel. Consider the first region. Dividing the rise of 4 meters for the run of 8 meters gives $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4\text{ m}}{8\text{ m}} = \frac{1}{2}$.



Now for a big discovery! In going from the slope function to the elevation function, the elevation function gives the running total of the area swept out by the slope function as we move from left to right.

Going from 0 to 8 sweeps out an area of 4. Note that at $x = 8$ we have $f(8) = 4$. Then no additional area is added from 8 to 12. So, we stay at 4 in the elevation function. The final leg of the hike has negative area which brings us down to the same elevation we started at for $y = f(x)$. Here is where we could lift the elevation function upward by a constant amount and the slope function would not change. This feature requires us to add an overall constant when we go from the slope function to the elevation function. Or, saying this in calculus terms, we add a constant when we integrate the slope function, i.e., when take the antiderivative of the slope function. We can also drop the slope descriptor. One typically says when we take the antiderivative of a function we add an overall constant with our result.

The first section of our course is finding derivatives for the functions you learned in algebra and trigonometry. In the second section we take antiderivatives, which we refer to as doing integrations. We refer to these two sections of our course as differential calculus and integral calculus. Below is an outline for our course.

0. Topics (This Chapter) - Syllabus
A. Functions
B. Basic Trig
C. The Derivative
D. Powers of x
E. The Chain Rule
F. The Addition & Subtractive Rules
G. The Product Rule
H. The Quotient Rule
I. The Sine
J. The Cosine and Tangent
K. Inverse Trig Functions
L. The Natural Exponential
M. The Natural Logarithm
N. The Hyperbolic Functions
O. The Inverse Hyperbolic Functions
P. Any Base a^x
Q. Implicit Differentiation
R. Max-Min Problems
S. Fundamental Theorem of Calculus
T. Integration by Simple Substitution
U. Trig Substitutions
V. Integration by Parts
W. Partial Fractions
X. Integration of Inverse Functions
Y. Integration in 2D
Z. Integration in 3D