

HW L1. A Trig Identity. Use Euler exponentials to show that $4 \cos^3 \theta = \cos(3\theta) + 3 \cos \theta$.



HW L2. The Cauchy Formula for Dispersion.

Augustin-Louis Cauchy (1789-1857). French mathematician, physicist, and engineer. Among Cauchy’s many achievements is his contributions in complex variables. In this problem we investigate his empirical formula for the index of refraction in a dispersive medium. One of his formulas is

$$n(\lambda) = A + \frac{B}{\lambda^2}, \text{ where } A \text{ and } B \text{ are constants for the medium. } A$$

popular crown glass is made by Schott, a glass company in Mainz, Germany. The company can be traced back to a company founded in 1884 by Otto Schott, Ernst Abbe of the Abbe number, Carl Zeiss and son Roderich Zeiss. The popular universal glass borosilicate glass BK7 is used in a variety of high-quality applications.

Index of Refraction for Borosilicate Glass Schott BK7 at Three Wavelengths			
Fraunhofer Line	F	D	C
Element	Hydrogen	Sodium	Hydrogen
Description	H-beta	Doublet Average	H-alpha
Color	Blue	Yellow	Red
Wavelength	486.1 nm	589.3 nm	656.3 nm
Refractive Index n	1.522	1.517	1.514
Cauchy Result for n	?	?	?

Complete the above table to four significant figures using the Cauchy approximation where the Cauchy constants for BK7 glass are $A = 1.5046$ and $B = 0.00420$ when the wavelengths entered into the formula are in microns (μm). Include the numerical steps for each of your calculations.

HW L3. Group Velocity. Derive the following group velocity formula in a dispersive medium

$$v_g = \frac{c}{n} \left[1 + \frac{\lambda}{n} \frac{dn}{d\lambda} \right] \text{ starting from } v_g = \frac{d\omega}{dk}.$$

HW L4. Phase Velocity and Group Velocity.

a) What is the phase velocity in outer space for light emitted from the 3-2 Balmer transition, i.e., 656.3 nm, to three significant figures in km/s? This beautiful deep-red light of emission nebulae is also designated as H-alpha, $H\alpha$, or $H-\alpha$.

b) Use your group velocity formula of HW L3 and the Cauchy dispersion formula of HW L2 in order to calculate the group velocity in km/s for $H\alpha$ in Borosilicate Glass Schott BK7 to three significant figures?